Monte Carlo Evaluation of Non-Linear Scattering Equations for Subsurface Scattering

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Graphics World-Views

- Geometry
  - Hidden surface elimination, shading

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- Rendering Equation
  - *Light transport problem*

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Graphics World-Views

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- Image-Based Rendering
  - *Light as a first class primitive*
- Scattering Equations
  - *Scattering as a first class primitive*

Breakthroughs in Transport Theory

- Equation of Transfer (1889–1913)
  - Lommel, Schuster, Schwarzwald, King

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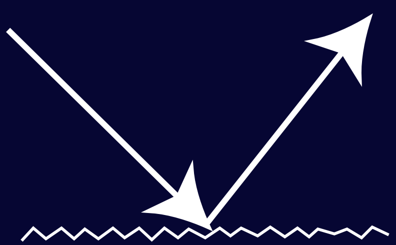
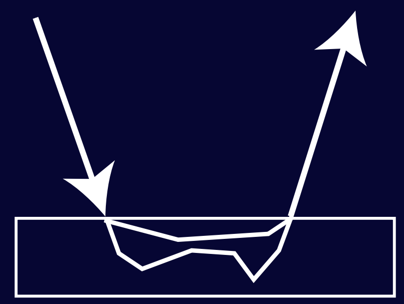
- Equation of Transfer (1889–1913)
  - Lommel, Schuster, Schwarzwald, King
- Invariance Principles (1942–1965)
  - Ambarzumian, Chandrasekhar, Preisendorfer

What is a Scattering Function?

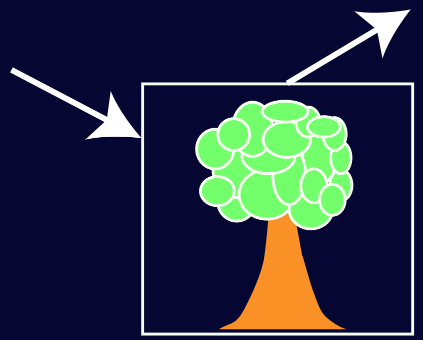
BRDF (4D)



BSSRDF (6D)



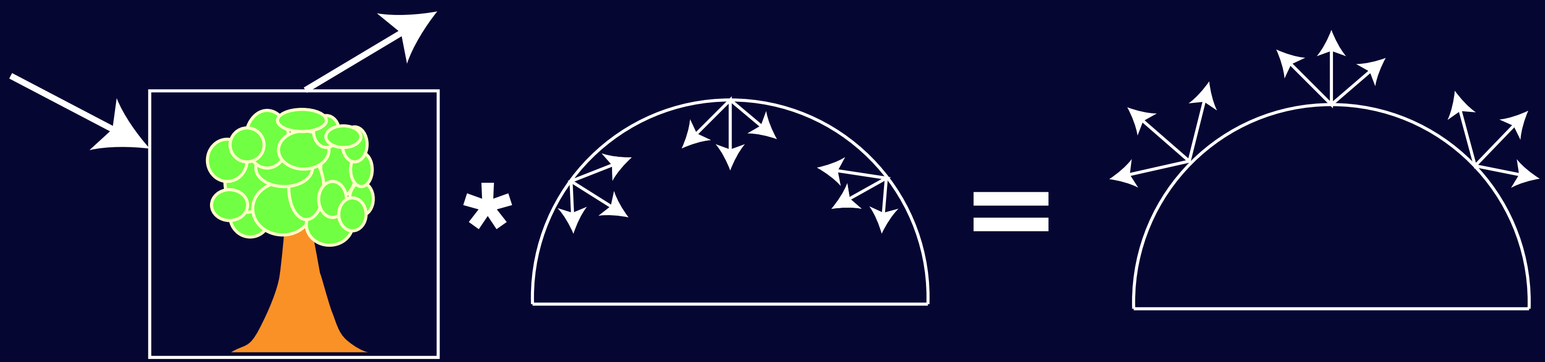
General (8D, ...)



Abstracts away lower-level scattering processes

What is a Scattering Function?

Maps incident radiance to exitant radiance



$$\int \mathrm{S}(r' \rightarrow r)\, \mathrm{L_i}(r')\, \mathrm{d}r' = \mathrm{L_o}(r)$$

Contributions

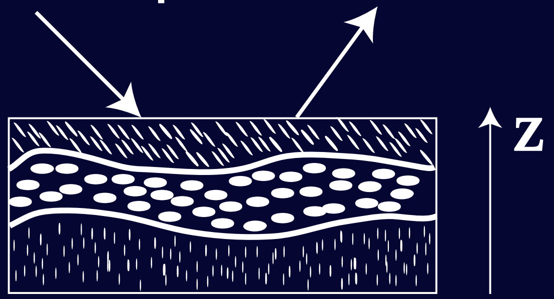
- Introduction to graphics
- Application to subsurface scattering
- 3D formulation
- Monte Carlo solutions

Overview

- Eqn of transfer vs. scattering equations
- Scattering equations
  - Adding equations
  - Differential form
- Monte Carlo solution
- Results

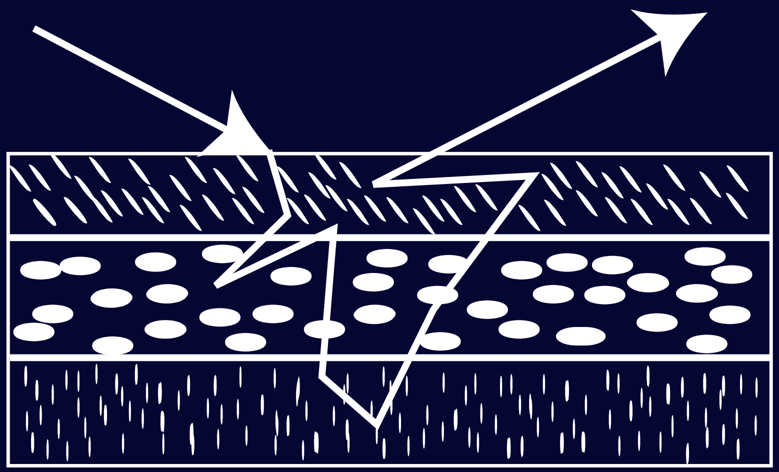
Subsurface Scattering

- Layered medium, z parameterization



- Attenuation coefficient σ_t
- Scattering coefficient σ_s
- Phase function $\mathrm{k}(\omega_i \rightarrow \omega_o)$
- 1D: BRDF, 3D: BSSRDF

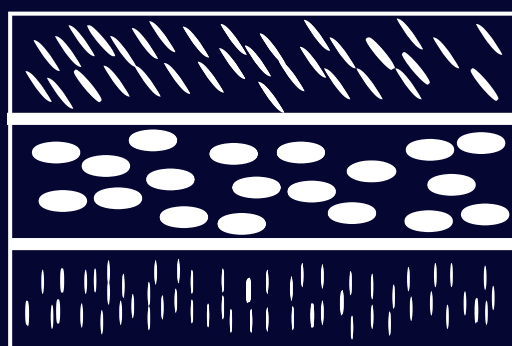
Scattering functions with the equation of transfer

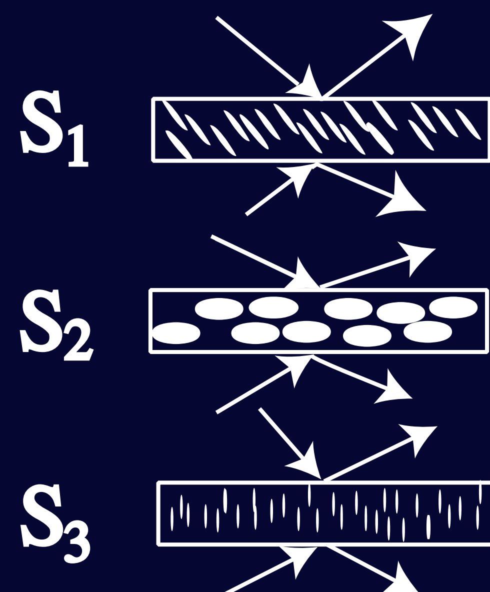


Westin *et al.*, Hanrahan & Krueger, Gondek *et al.*

A New Approach

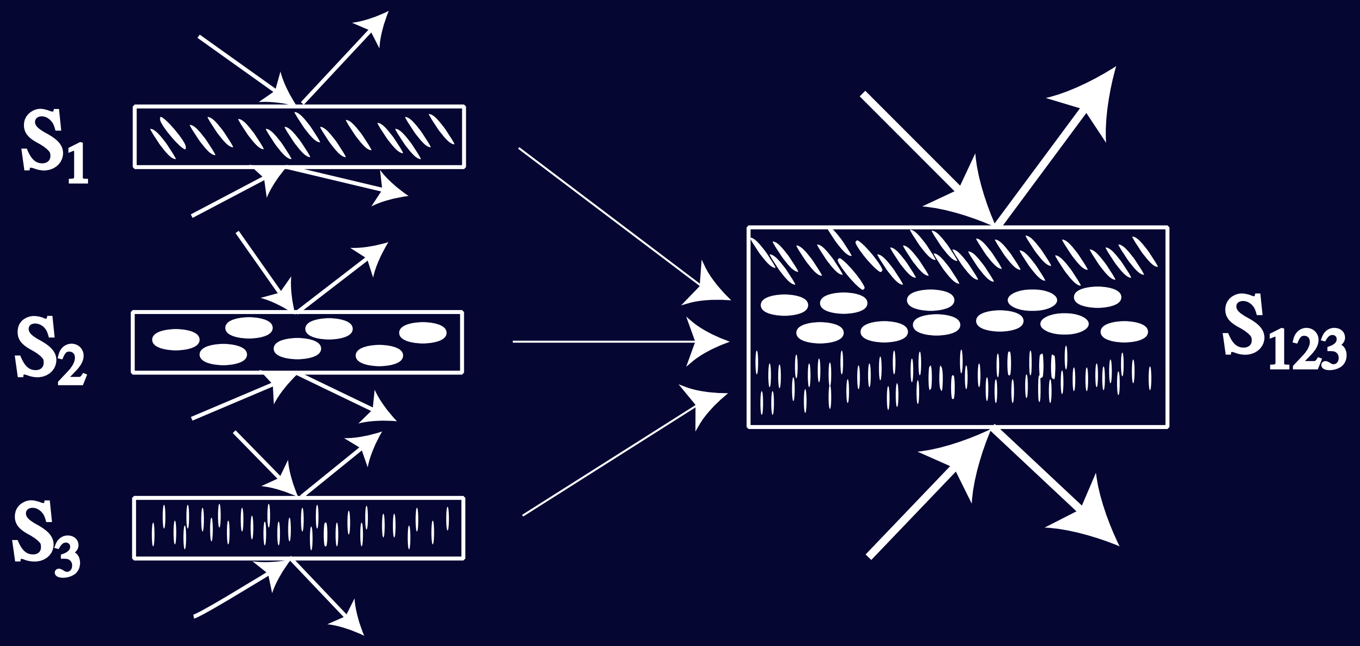
Scattering functions completely describe layers



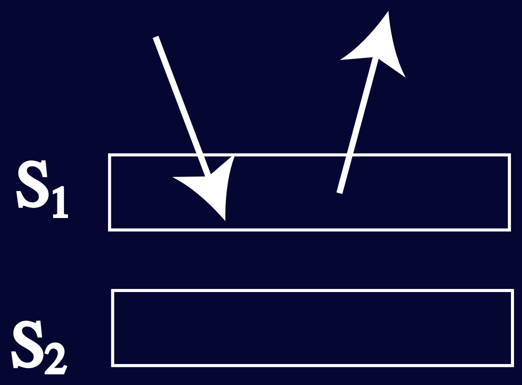


A New Approach

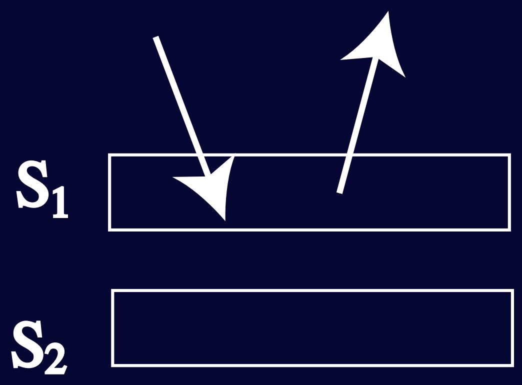
Adding equations build up new scattering functions



Adding Equations

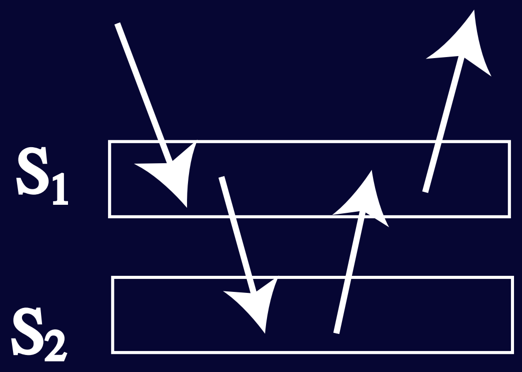


Adding Equations



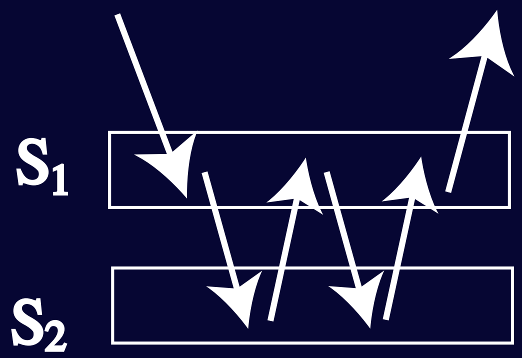
$$\mathbf{S}_{12} = \mathbf{S}_1 + \cdots$$

Adding Equations



$$\mathbf{S}_{12} = \mathbf{S}_1 + \mathbf{S}_1\mathbf{S}_2\mathbf{S}_1 + \cdots$$

Adding Equations



$$\mathrm{S}_{12} = \mathrm{S}_1 + \mathrm{S}_1\mathrm{S}_2\mathrm{S}_1 + \mathrm{S}_1\mathrm{S}_2\mathrm{S}_1\mathrm{S}_2\mathrm{S}_1 + \cdots$$

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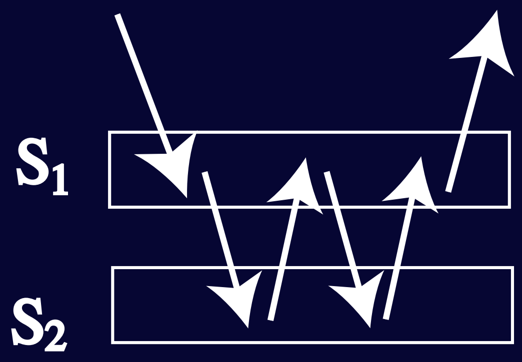


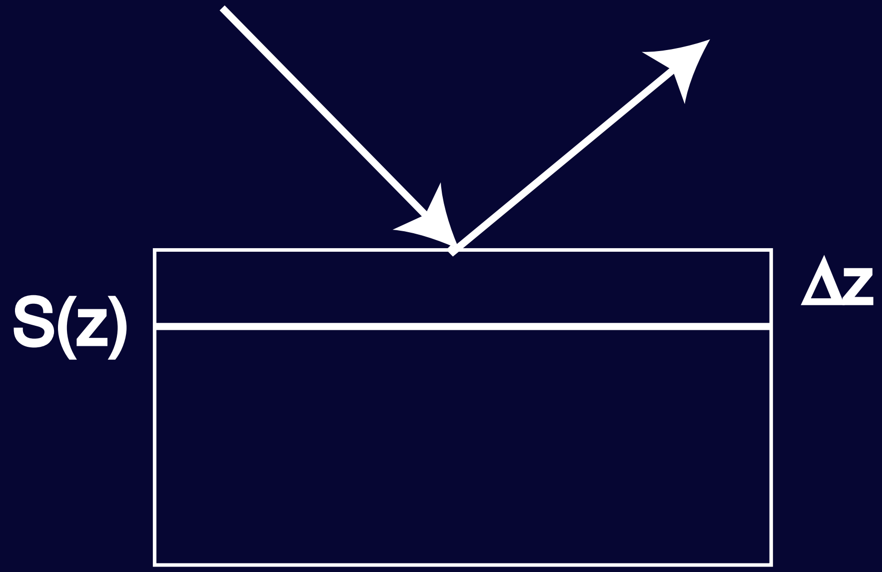
$$\mathbf{S}_{12} = \mathbf{S}_1 + \mathbf{S}_1\mathbf{S}_2\mathbf{S}_1 + \mathbf{S}_1\mathbf{S}_2\mathbf{S}_1\mathbf{S}_2\mathbf{S}_1 + \cdots$$

$$\downarrow$$

$$\iint \mathrm{S}_1(r_i \to r')\mathrm{S}_2(r' \to r'')\mathrm{S}_1(r'' \to r_o)\mathrm{d}r'\mathrm{d}r''$$

Integro-differential Scattering Eqn

$$\frac{\partial \mathrm{S}}{\partial z} = -\alpha \mathrm{S}(z) + \mathbf{k} + \mathbf{k}\mathrm{S}(z) + \mathrm{S}(z)\mathbf{k} + \mathrm{S}(z)\mathbf{k}\mathrm{S}(z)$$



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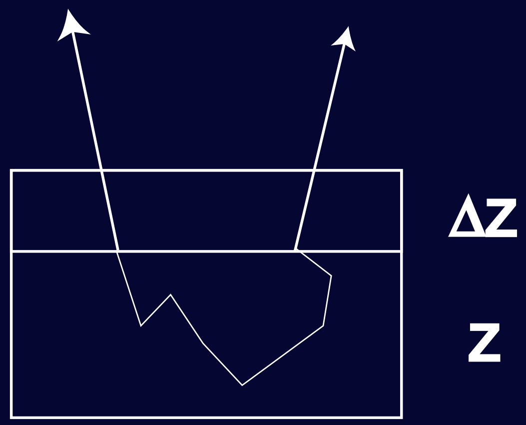

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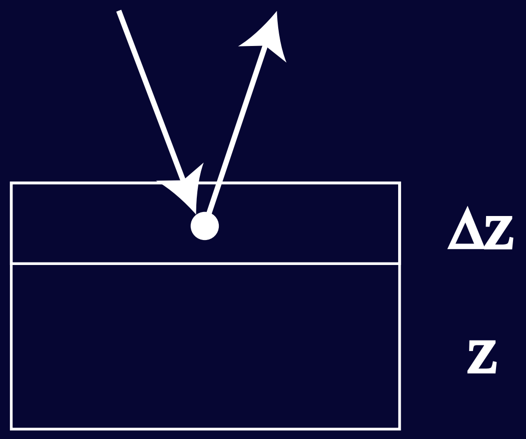
$$\mathbf{k} = \mathrm{k}(\omega_i \rightarrow \omega_o)$$



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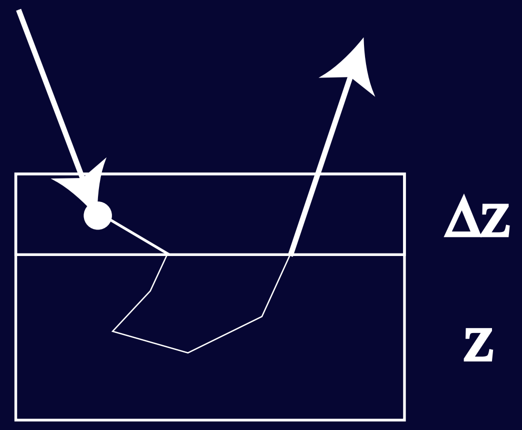
$$\mathbf{k}\mathrm{S}(z) = \int \mathrm{k}(\omega_i \rightarrow -\omega')\mathrm{S}(z, \omega' \rightarrow \omega_o)\frac{\mathrm{d}\omega'}{\cos\omega'}$$



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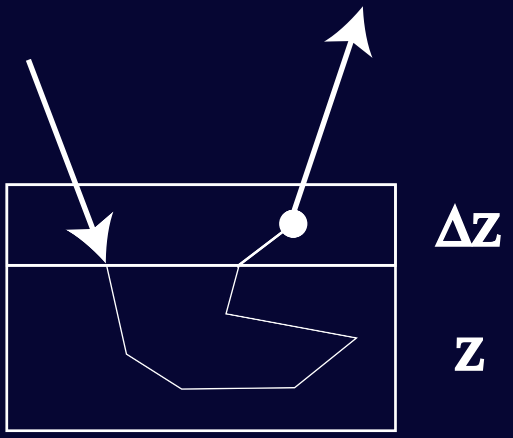

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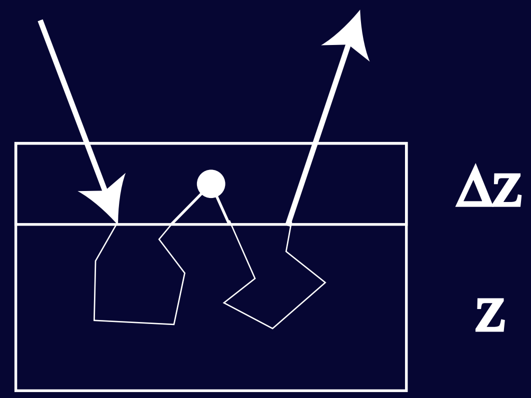
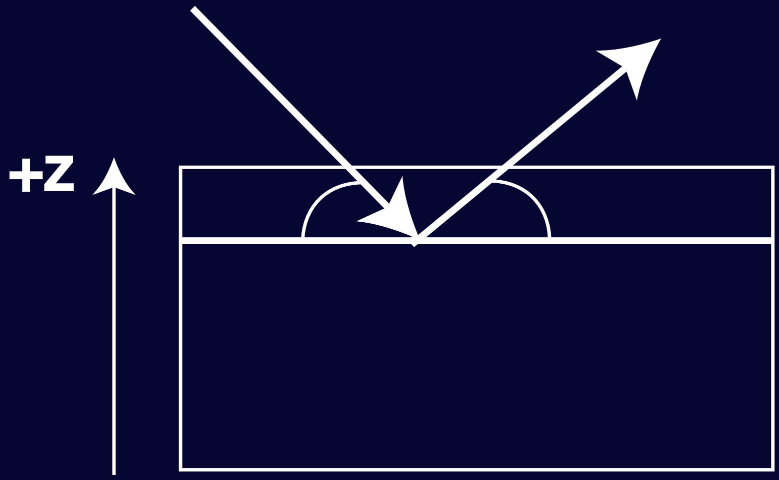


Integral Scattering Equation

$$\mathrm{S}(z) = \int_0^z e^{-\tau}(\mathbf{k} + \mathbf{k}\,\mathrm{S}(z') + \mathrm{S}(z')\,\mathbf{k} + \mathrm{S}(z')\,\mathbf{k}\,\mathrm{S}(z'))\mathrm{d}z'$$

τ is the optical path length traveled by the ray



Random Walk Monte Carlo

$$\mathrm{S}(z, \omega_i \rightarrow \omega_o) = \int_0^z \mathrm{e}^{-\tau} (\mathbf{k} + \mathbf{kS} + \mathbf{Sk} + \mathbf{SkS}) \, \mathrm{d}z'$$

Random Walk Monte Carlo

$$\mathrm{S}(z, \omega_i \rightarrow \omega_o) = \int_0^z \mathrm{e}^{-\tau}(\mathbf{k} + \mathbf{kS} + \mathbf{Sk} + \mathbf{SkS})\,\mathrm{d}z'$$

- Sample z', $(0 < z' < z)$

Random Walk Monte Carlo

$$\mathrm{S}(z, \omega_i \longrightarrow \omega_o) = \int_0^z \mathrm{e}^{-\tau} (\mathbf{k} + \mathbf{kS} + \mathbf{Sk} + \mathbf{SkS})\, \mathrm{d}z'$$

- Sample z', $(0 < z' < z)$
- MC estimate is

$$\frac{\mathrm{e}^{-\tau}}{\mathrm{pdf}(z')} (\mathbf{k} + \mathbf{kS} + \mathbf{Sk} + \mathbf{SkS})$$

Random Walk Monte Carlo

$$\mathrm{S}(z, \omega_i \to \omega_o) = E[w(\mathbf{k} + \mathbf{kS} + \mathbf{Sk} + \mathbf{SkS})]$$

$$w\mathbf{k} = w\,\mathrm{k}(z', \omega_i \to \omega_o)$$

Random Walk Monte Carlo

$$\mathrm{S}(z, \omega_i \to \omega_o) = E[w(\mathbf{k} + \mathbf{kS} + \mathbf{Sk} + \mathbf{SkS})]$$

$$w\mathbf{kS} =$$

$$\frac{w}{4\pi} \boxed{\int_\Omega \mathrm{k}(z', -\omega' \to \omega_o)\, \mathrm{S}(z', \omega_i \to \omega')} \frac{\mathrm{d}\omega'}{\cos\, \omega'}$$

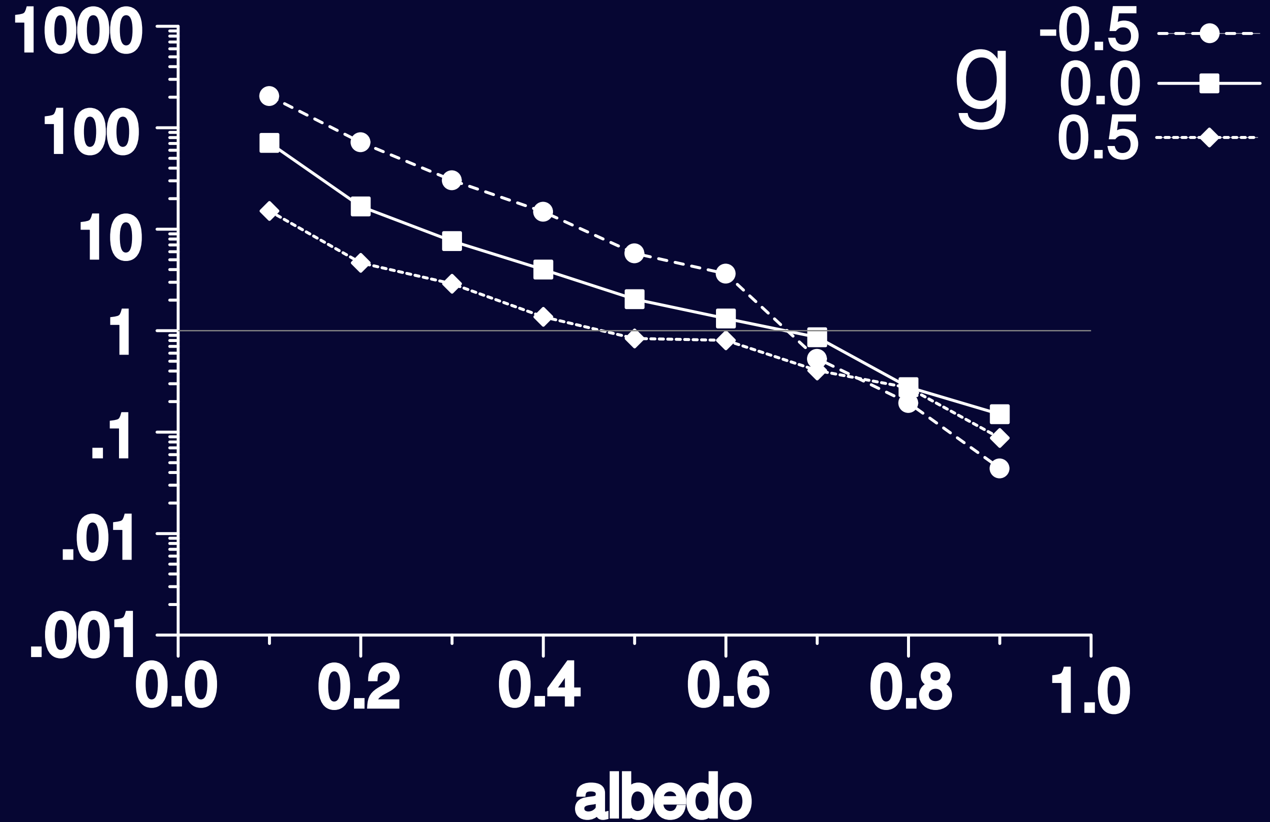
Key Characteristics

- Problem becomes simpler ($z' < z$)
  - Divide and conquer
  - Leads to *iterative* solutions
- Scattering function tends to be smooth
  - Multiple scattering $\longrightarrow$ isotropic distributions
  - New importance sampling opportunities

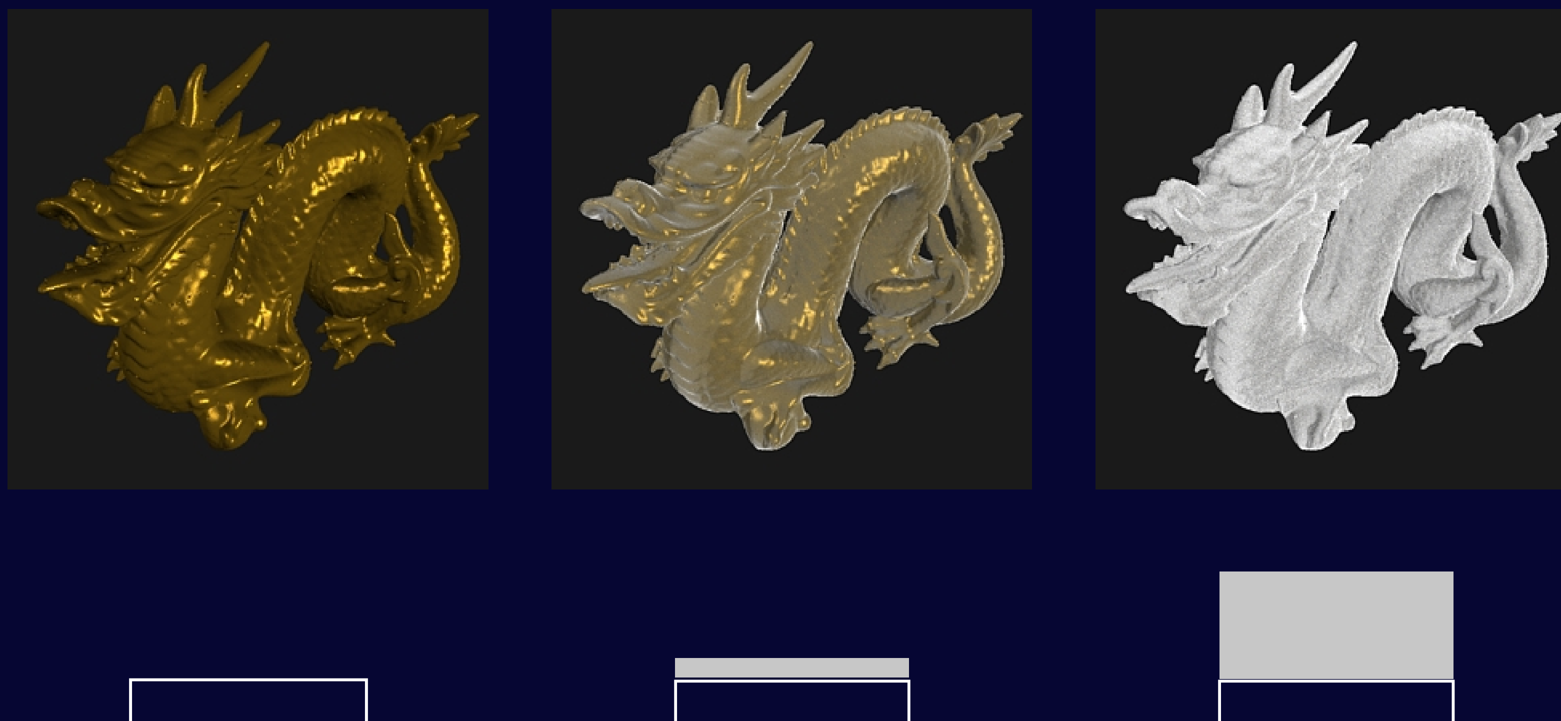
Efficiency

- Scattering from a homogeneous slab
  - Classic problem in astrophysics: "the standard problem"
- Compared efficiency to equation of transfer

Efficiency

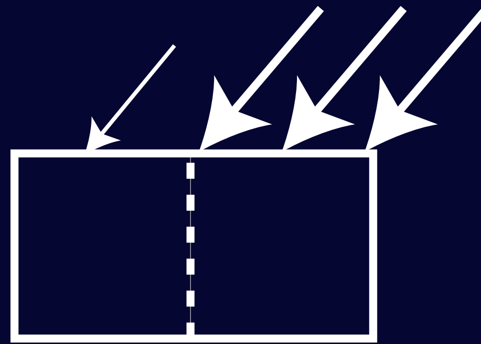


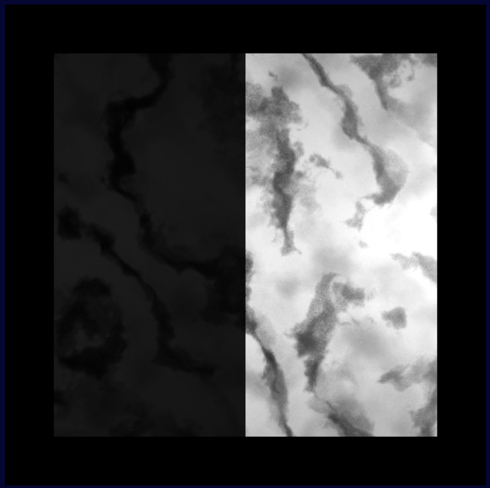
Dusty Dragon



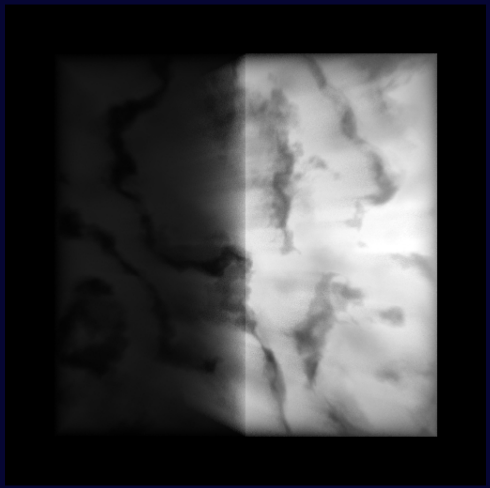
Dusty Dragon

Marble Slab





1D Scattering Eqn



3D Scattering Eqn

Summary

- Scattering equations
  - New theoretical framework for rendering
  - Two forms: differential and adding
- Monte Carlo Solution
  - Application to complex problems
  - New sampling methods

Future Work

- Importance sampling/different MC methods
- Extend to completely general setting
  - Different parameterizations of space
  - Geometry
  - Hierarchical approaches:
    the interaction principle
- Level of detail, accurate imposters

Acknowledgements

- Eric Veach
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