Written assignment #1 - sampling and antialiasing

Due in class Tuesday, April 28

(1) Prove that convolution is associative, i.e. that

$$[f(x) * g(x)] * h(x) = f(x) * [g(x) * h(x)].$$

(2) The triangle function of width 2 and unit area is frequently used as a filter in computer graphics. It is defined as:

$$\Lambda(x) = \begin{cases} 1 + x & -1 \le x < 0 \\ 1 - x & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Using the definition of convolution, show that the convolution of two rect functions yields the triangle function, i.e. that $\Pi(x) * \Pi(x) = \Lambda(x)$.

(b) Prove that reconstructing a sampled 1D signal by convolving it with the triangle function $\Lambda(x)$ corresponds to linearly interpolating its samples. Assume that the samples are taken one unit apart.

(3) Intensity I on a CRT is typically related to frame buffer pixel value V by the power function $I = V^{\gamma}$ for some value of gamma. If we antialias a polygon edge using one of the methods described in class and display it on a CRT without correcting for this power function, typically by loading an inverse power function into the hardware colormap, then the image will be visually unsatisfactory.

(a) What will the artifacts look like? Sketch them.

(b) The presence of an uncorrected power function violates one of the premises underlying the sampling and reconstruction pipeline presented in class. Which one?

(4) Unless dithering is used, quantization using too few bits produces visual artifacts. In particular, smoothly shaded surfaces exhibit contours. Are these artifacts a form of aliasing? Defend your answer.

(5) A *telecine chain* converts a movie into a television program essentially by pointing a television camera at the image produced by a movie projector. This implies changing the temporal sampling rate from 24 frames per second to 60 frames per second. (Let's assume non-interlaced video running at 60 frames per second.) How might such a device perform the required change in frame rate? Explain your solution in terms of the resampling pipeline described in class. Identify the reconstruction and bandlimiting filters. How well would you expect your solution to work? There is more than one possible answer here. Your solution needn't match the way real telecine chains work, but it should be plausible. (Note that modern CCD-based cameras don't employ raster scanning; all pixels "sense" the incoming light at once. This should simplify your analysis.)

(6) **For extra credit:** Why are the scanlines in an interlaced television system like NTSC horizontal?