The Light Field

Light field = radiance function on rays

Conservation of radiance

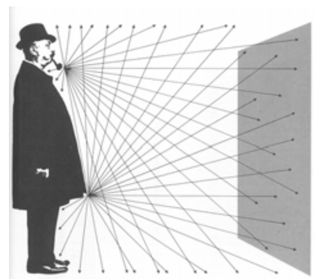
Measurement equation

Throughput and counting rays

Conservation of throughput

Area sources and irradiance

Form factors and radiosity

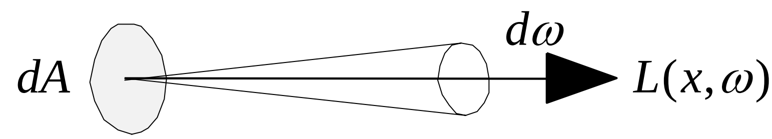


From London and Upton

Light Field = Radiance(Ray)

Field Radiance

Definition: The field *radiance* (*luminance*) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction



Radiance is the quantity associated with a ray

Light Probe ⇒ Environment Map

$$L(x, y, z, \theta, \varphi)$$

Miller and Hoffman, 1984

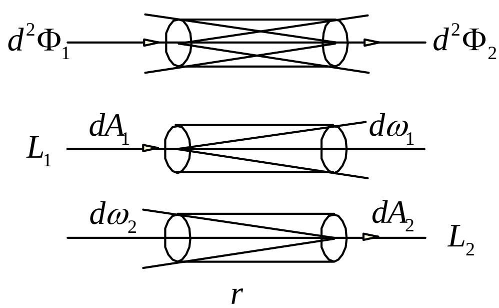
Properties of Radiance

Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   - ∴ Radiance is a function on rays
   - ∴ All other field quantities are derived from it
2. Radiance invariant along a ray.
   - ∴ 5D ray space reduces to 4D
3. Response of a sensor proportional to radiance.

1st Law: Conversation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates



$$d^2\Phi_1 = d^2\Phi_2$$

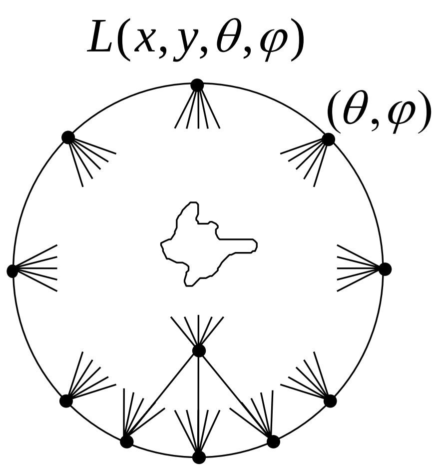
$$d^2\Phi_1 = L_1 d\omega_1 dA_1$$

$$d^2\Phi_2 = L_2 d\omega_2 dA_2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

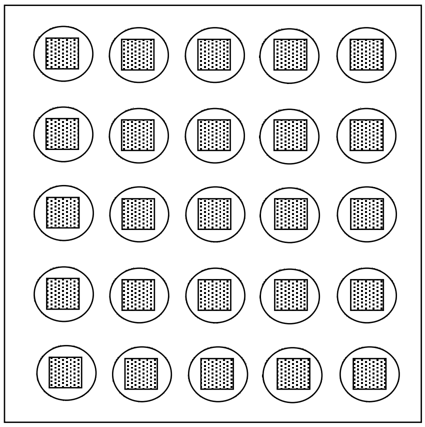
$$\therefore L_1 = L_2$$

Spherical Gantry ⇒ 4D Light Field



Capture all the light leaving an object - like a hologram

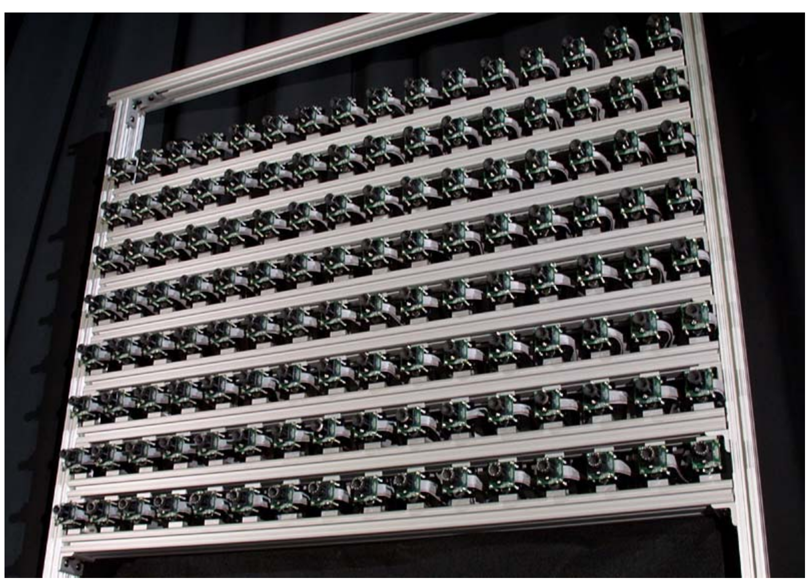
Two-Plane Light Field



2D Array of Cameras **2D Array of Images**

$$L(u,v,s,t)$$

Multi-Camera Array ⇒ Light Field

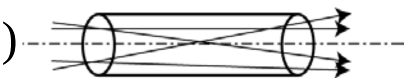


Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

$$r(u_1, v_1, u_2, v_2)$$

$dA_1(u_1, v_1)$



$dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1 dA_2}{\left| x_1 - x_2 \right|^2}$$

T **measures/count the number of rays in the beam**

Conservation of Throughput

- **Throughput conserved during propagation**
  - **Number of rays conserved**
  - **Assuming no attenuation or scattering**
- **n^2 (index of refraction) times throughput invariant under the laws of geometric optics**
  - **Reflection at an interface**
  - **Refraction at an interface**
    - **Causes rays to bend (kink)**
  - **Continuously varying index of refraction**
    - **Causes rays to curve; mirages**

Conservation of Radiance

Radiance is the ratio of two quantities:

1. Power

2. Throughput

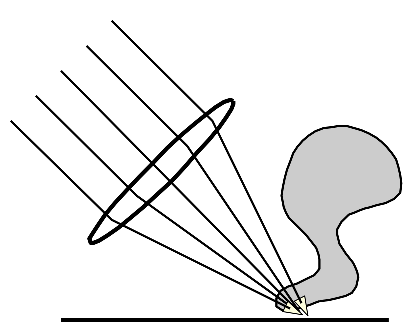
$$L(r) = \lim_{\Delta T \to 0} \frac{\Delta\Phi(\Delta T)}{\Delta T} = \frac{d\Phi}{dT}$$

Since power and throughput are conserved,

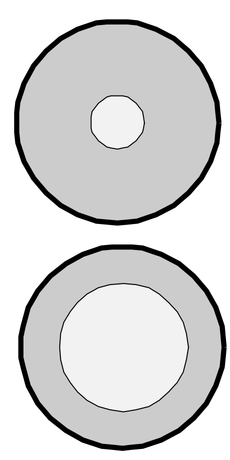
$\therefore$ **Radiance conserved**

Quiz

Does radiance increase under a magnifying glass?

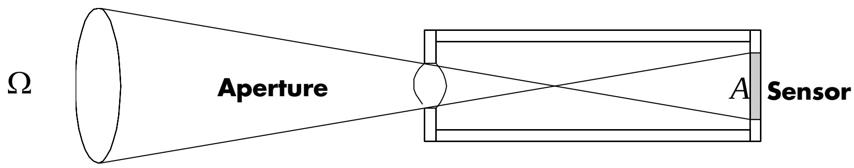


No!!



Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.



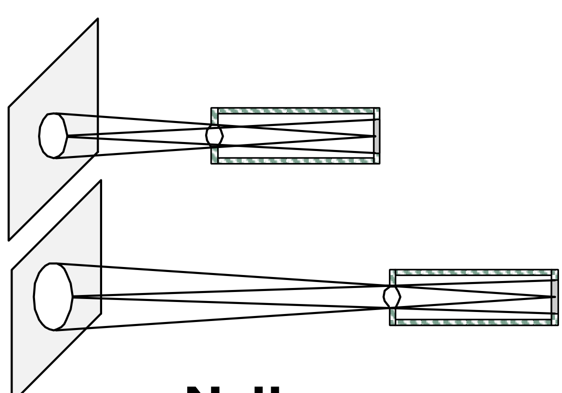
$$R = \int_A \int_\Omega L\, d\omega\, dA = \bar{L} T \qquad T = \int_A \int_\Omega d\omega\, dA$$

L is what should be computed and displayed.

T quantifies the gathering power of the device; the higher the throughput the greater the amount of light gathered

Quiz

Does the brightness that a wall appears to the sensor depend on the distance?



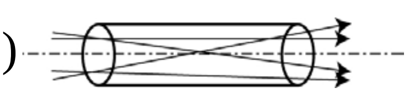
No!!

Measuring Rays = Throughput

Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

$$r(u_1, v_1, u_2, v_2)$$

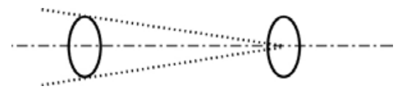
$dA_1(u_1, v_1)$  $dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1 dA_2}{\left|x_1 - x_2\right|^2}$$

Measure/count the number of rays in the beam

Parameterizing Rays

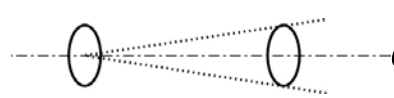
Parameterize rays wrt to receiver $r(u_2, v_2, \theta_2, \phi_2)$

$d\omega_2(\theta_2, \phi_2)$  $dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2$$

Parameterizing Rays

Parameterize rays wrt to source $r(u_1, v_1, \theta_1, \phi_1)$

$dA_1(u_1, v_1)$  $d\omega_1(\theta_1, \phi_1)$

$$d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1$$

Parameterizing Rays

Tilting the surfaces reparameterizes the rays

$r(u_1, v_1, u_2, v_2)$

$d\vec{A}_1(u_1, v_1)$  $d\vec{A}_2(u_2, v_2)$

$$d^2T = \frac{\cos\theta_1 \cos\theta_2}{\left|x_1 - x_2\right|^2} dA_1\, dA_2$$

$$= d\vec{\omega}_1 \bullet d\vec{A}_1$$

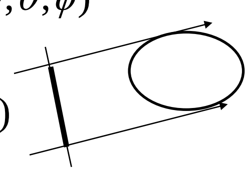
$$= d\vec{\omega}_2 \bullet d\vec{A}_2$$

All these throughputs must be equal.

Parameterizing Rays: $S^2 \times R^2$

Parameterize rays by $r(x, y, \theta, \phi)$

Projected area $\tilde{A}(\vec{\omega})$



$\vec{\omega}$

Measuring the number or rays that hit a shape

$$T = \int_{S^2} d\omega(\theta,\varphi) \int_{R^2} dA(x,y)$$

$$= \int_{S^2} \tilde{A}(\theta,\varphi)\, d\omega(\theta,\varphi)$$

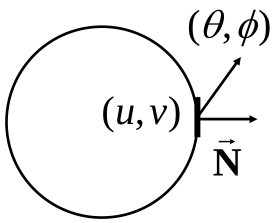
$$= 4\pi \bar{\tilde{A}}$$

Sphere:

$$T = 4\pi \bar{\tilde{A}} = 4\pi^2 R^2$$

Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u, v, \theta, \phi)$



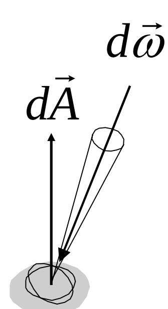
$$T = \underbrace{\left[\int_{M^2} dA(u,v) \right]}_{S} \underbrace{\left[\int_{H^2(\vec{\mathbf{N}})} \cos\theta \, d\omega(\theta, \varphi) \right]}_{\pi}$$

Sphere: $T = \pi S = 4\pi^2 R^2$

Crofton's Theorem: $4\pi \bar{\tilde{A}} = \pi S \Rightarrow \bar{\tilde{A}} = \frac{S}{4}$

Incident Surface Radiance

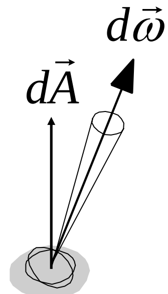
<u>Definition</u>: The incoming surface *radiance* (*luminance*) is the power per unit solid angle per unit projected area arriving at a receiving surface



$$L_i(x, \omega) \equiv \frac{d^2 \Phi_i(x, \omega)}{d\vec{\omega} \bullet d\vec{A}}$$

Exitant Surface Radiance

Definition: The outgoing surface radiance (luminance) is the power per unit solid angle per unit projected area leaving at surface



$$L_o(x,\omega) \equiv \frac{d^2\Phi_o(x,\omega)}{d\vec{\omega} \bullet d\vec{A}}$$

Alternatively: the intensity per unit projected area leaving a surface

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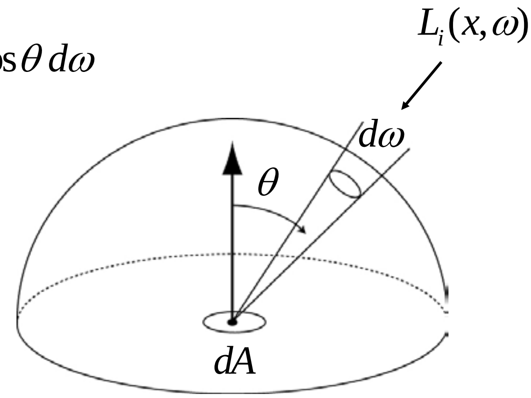
Irradiance from a Uniform Area Source

Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta\, dA\, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta\, d\omega$$



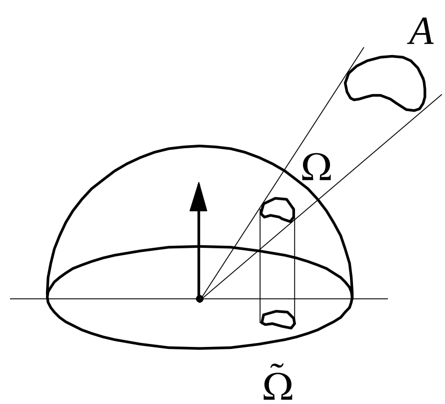
$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta\, d\omega$$

Uniform Area Source

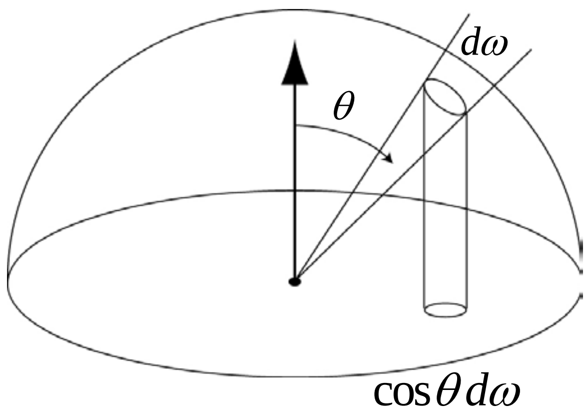
$$E(x) = \int_{H^2} L\cos\theta d\omega$$

$$= L\int_{\Omega} \cos\theta d\omega$$

$$= L\tilde{\Omega}$$



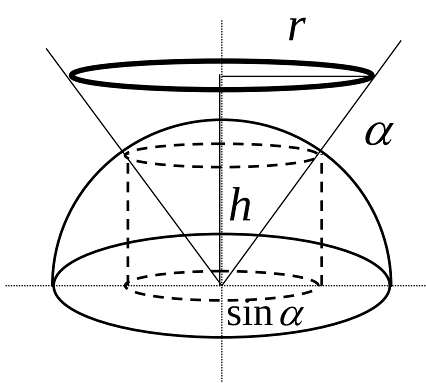
Projected Solid Angle



$$\int_{H^2} \cos\theta\, d\omega = \pi$$

Uniform Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_1^{\cos\alpha} \int_0^{2\pi} \cos\theta\, d\phi\, d\cos\theta$$

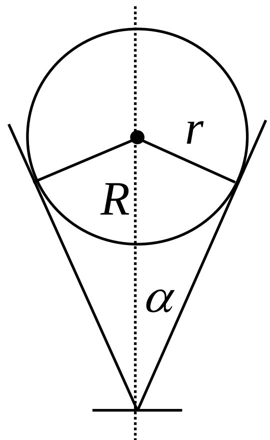
$$= 2\pi \left. \frac{\cos^2\theta}{2} \right|_1^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

Spherical Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int \cos\theta \, d\omega$$
$$= \pi \sin^2 \alpha$$
$$= \pi \frac{r^2}{R^2}$$

The Sun

Solar constant (normal incidence at zenith)

- **Irradiance** **1353 W/m²**
- **Illuminance** **127,500 lm/m² = 127.5 kilolux**

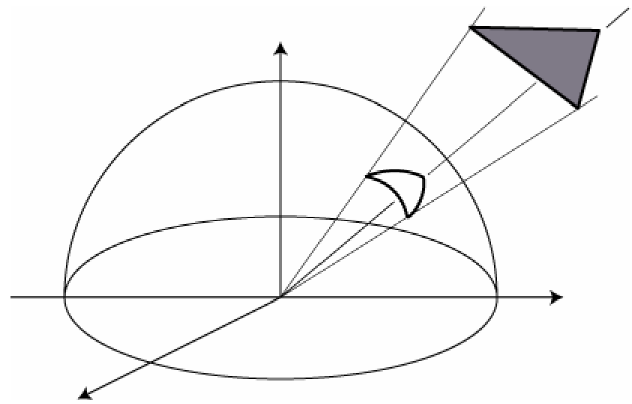
Solar angle

- α **= .25 degrees = .004 radians (half angle)**
- $\tilde{\Omega} = \pi \sin^2 \alpha \approx \pi\alpha^2$ **= 6 x 10^{-5} steradians**

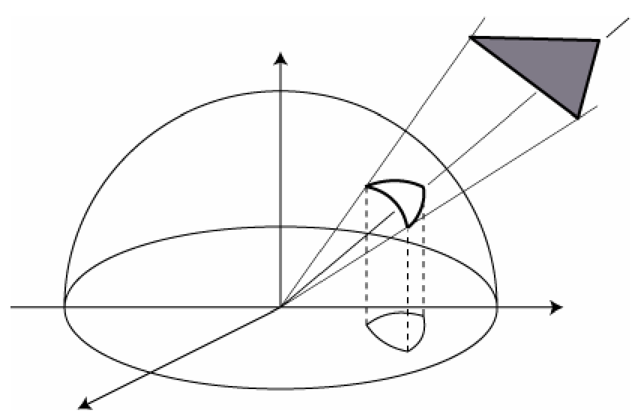
Solar radiance

$$L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \, W/m^2}{6 \times 10^{-5} \, sr} = 2.25 \times 10^7 \frac{\mathrm{W}}{m^2 \cdot sr}$$

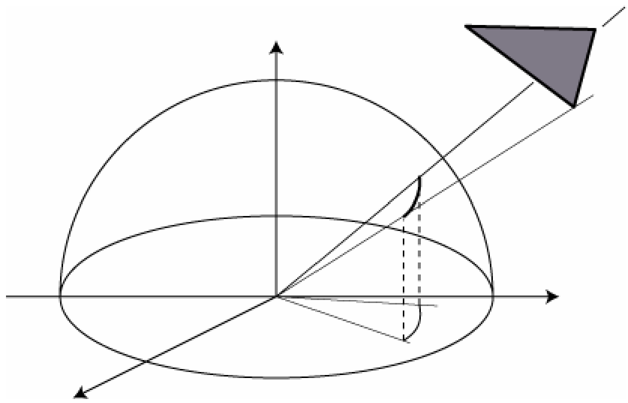
Polygonal Source



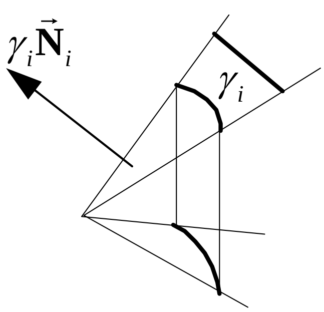
Polygonal Source

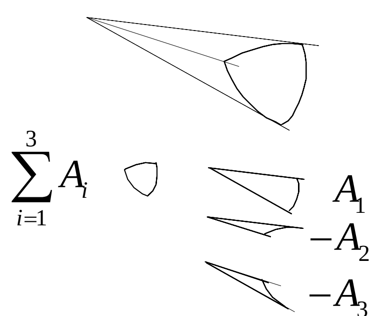


Polygonal Source



Lambert's Formula

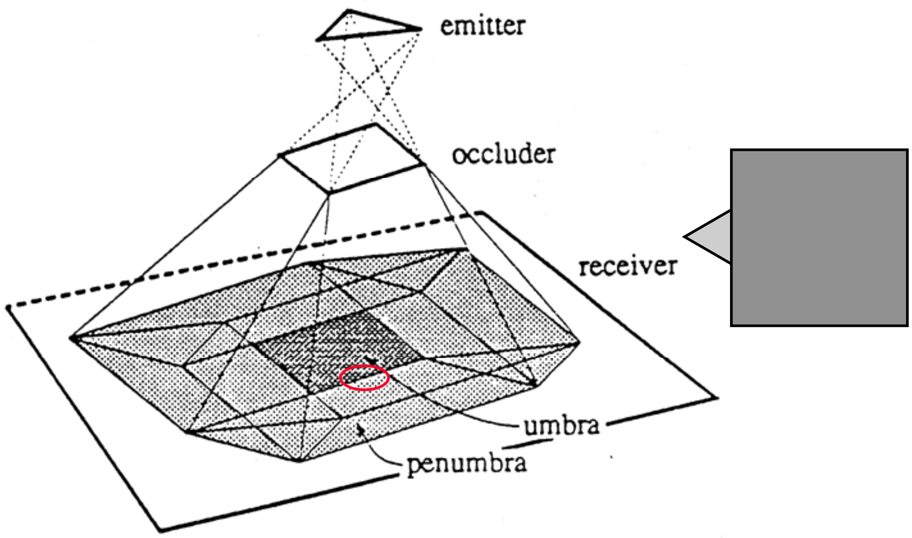




$$A_i = \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

Penumbras and Umbras



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Form Factors

Types of Throughput

1. Infinitesimal beam of rays (radiance)

$$d^2T(dA, dA') \equiv \frac{\cos\theta\cos\theta'}{|x - x'|^2} dA(x)\, dA(x')$$

2. Infinitesimal-finite beam (irradiance calc.)

$$dT(dA, A')\, dA \equiv \left[\int_{M'} \frac{\cos\theta\cos\theta'}{|x - x'|^2} dA(x') \right] dA(x)$$

3. Finite-finite beam (radiosity calc.)

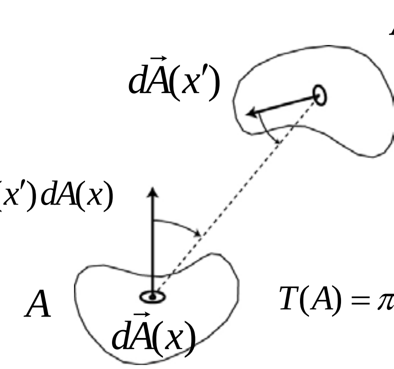
$$T(A, A') \equiv \int_M \int_{M'} \frac{\cos\theta\cos\theta'}{|x - x'|^2} dA(x')\, dA(x)$$

Probability of Ray Intersection

Probability of a ray hitting *A′* given that it hits *A*

$$\Pr(A' \mid A) = \frac{T(A', A)}{T(A)} = \frac{T(A', A)}{\pi A}$$

$$T(A', A) = \int_M \int_{M'} \frac{\cos\theta\cos\theta'}{|x - x'|^2} dA(x')\, dA(x)$$



$$T(A) = \pi A$$

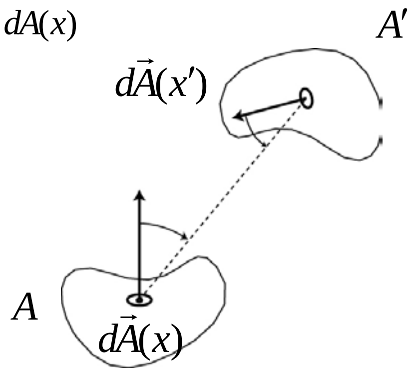
Another Formulation

$$T(A',A) = \int_M \int_{M'} \frac{\cos\theta \cos\theta'}{|x-x'|^2} dA(x') dA(x)$$

$$= \pi \int_M \int_{M'} G(x,x') dA(x') dA(x)$$

$$G(x,x') \equiv \frac{\cos\theta \cos\theta'}{\pi |x-x'|^2} V(x,x')$$

$$V(x,x') = \begin{cases} 0 & \neg visible \\ 1 & visible \end{cases}$$



Form Factor

Probability of a ray hitting *A′* given it hits *A*

$$\Pr(A' \mid A)\ T(A) = \Pr(A \mid A')\ T(A) = T(A',A)$$

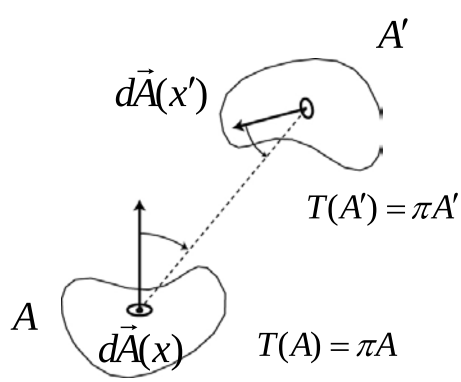
Form factor definition

$$F(A',A) = \Pr(A' \mid A)$$

$$F(A,A') = \Pr(A \mid A')$$

Form factor reciprocity

$$F(A',A)A = F(A,A')A'$$



Radiosity

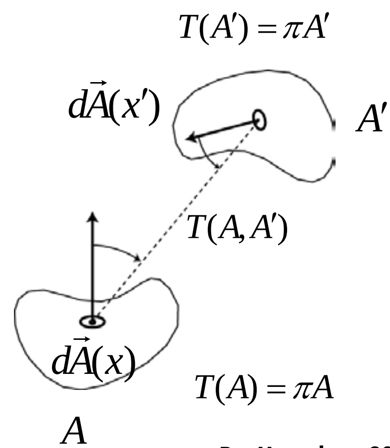
Power transfer from a constant radiance source

$$\Phi(A,A') = L(A')T(A,A')$$

$$= L(A')T(A')\frac{T(A,A')}{T(A')}$$

$$= \Phi(A')F(A,A')$$

$$E(A) = B(A')F(A',A)$$



Set up system of equations representing power transfers between objects

Form Factors and Throughput

Form factors represent the probability of ray leaving a surface intersecting another surface

- **Only a function of surface geometry**

Differential form factor

- **Irradiance calculations**

Form factors

- **Radiosity calculations (energy balance)**