Ray Tracing

Today

- Basic algorithms
- Overview of pbrt
- Ray-surface intersection for single surface

Next lecture

- Acceleration techniques for ray tracing large numbers of geometric primitives

Classic Ray Tracing

Greeks: Do light rays proceed from the eye to the light, or from the light to the eye?

Gauss: Rays through lenses

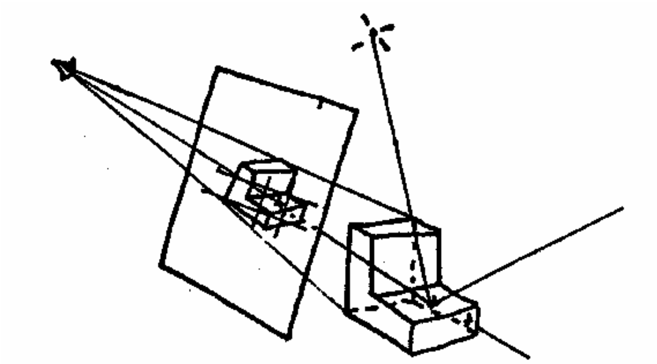
Three ideas about light

1. Light rays travel in straight lines
2. Light rays do not interfere with each other if they cross
3. Light rays travel from the light sources to the eye (but the physics is invariant under path reversal - reciprocity).

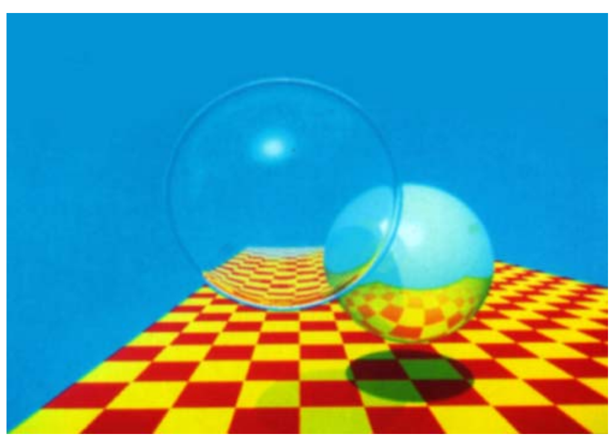
Ray Tracing in Computer Graphics

Appel 1968 - Ray casting

1. Generate an image by sending one ray per pixel
2. Check for shadows by sending a ray to the light



Ray Tracing in Computer Graphics



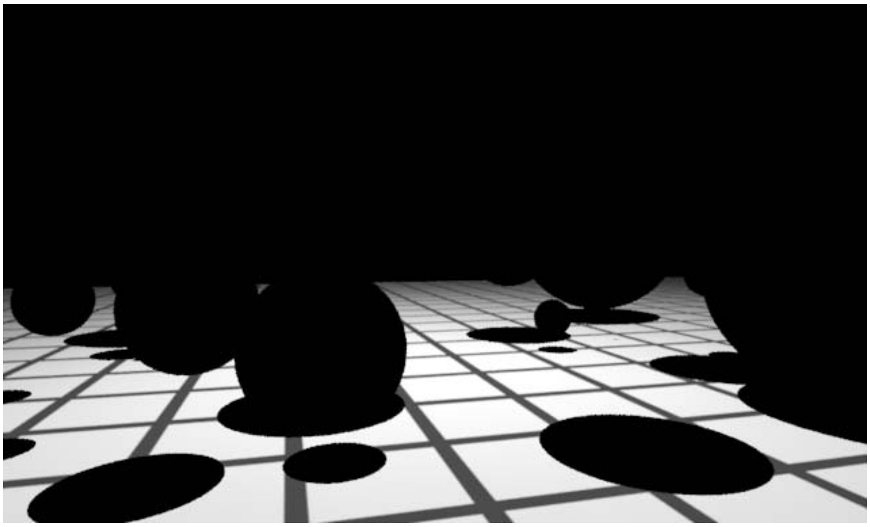
Whitted 1979

Recursive ray tracing (reflection and refraction)

Ray Tracing Video

Spheres-over-plane.pbrt (g/m 10)

Spheres-over-plane.pbrt (mirror 0)



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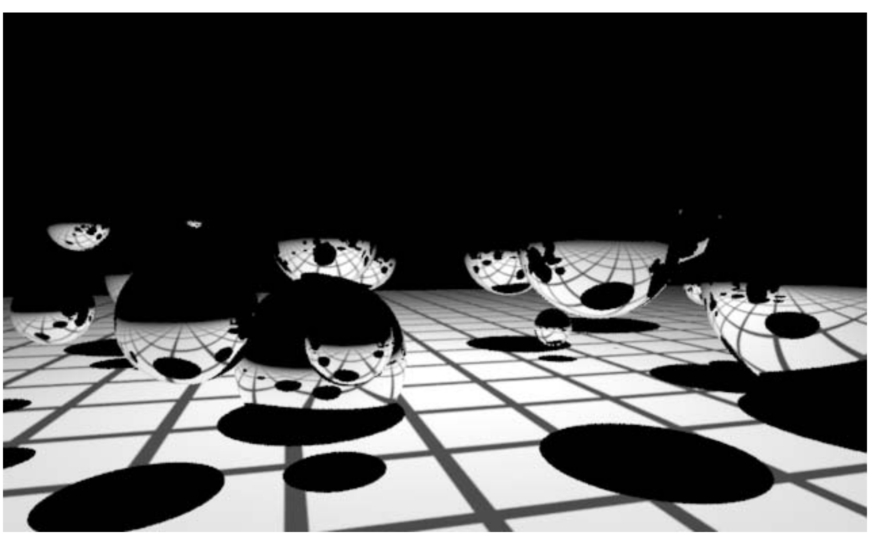
Spheres-over-plane.pbrt (mirror 1)



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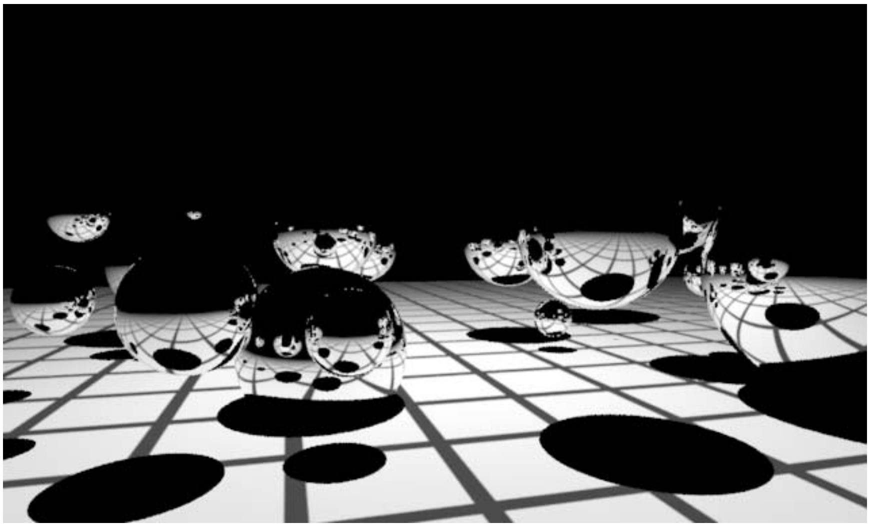
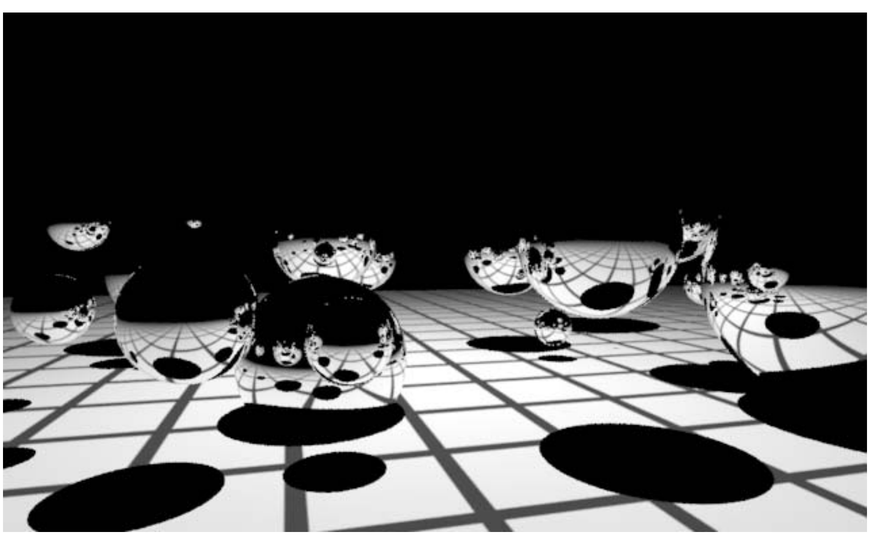
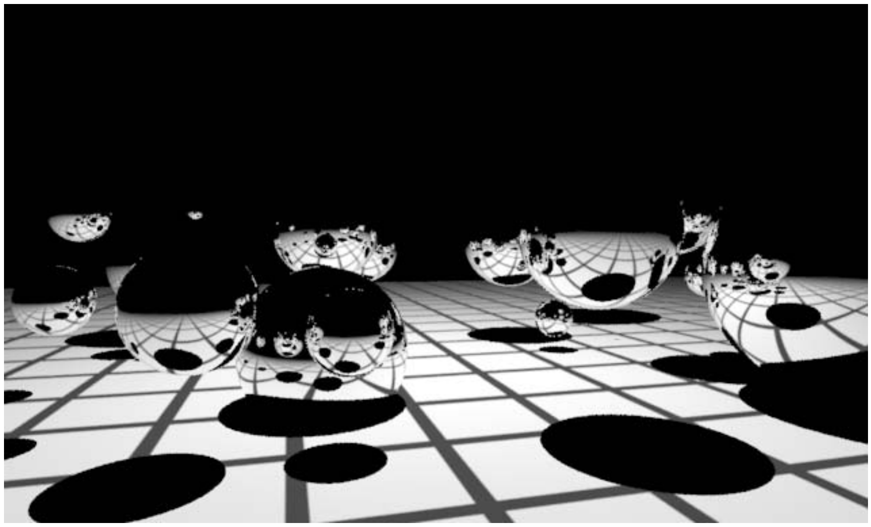
Spheres-over-plane.pbrt (mirror 2)



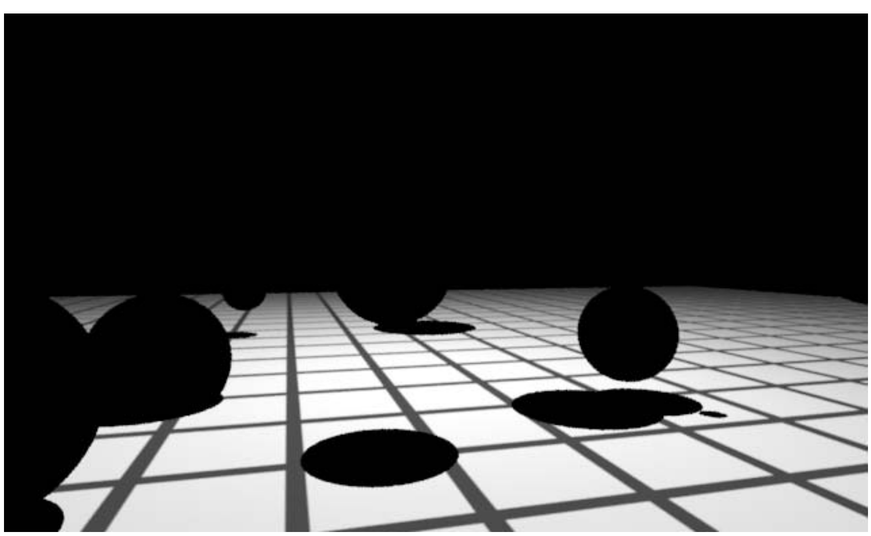
Spheres-over-plane.pbrt (mirror 5)



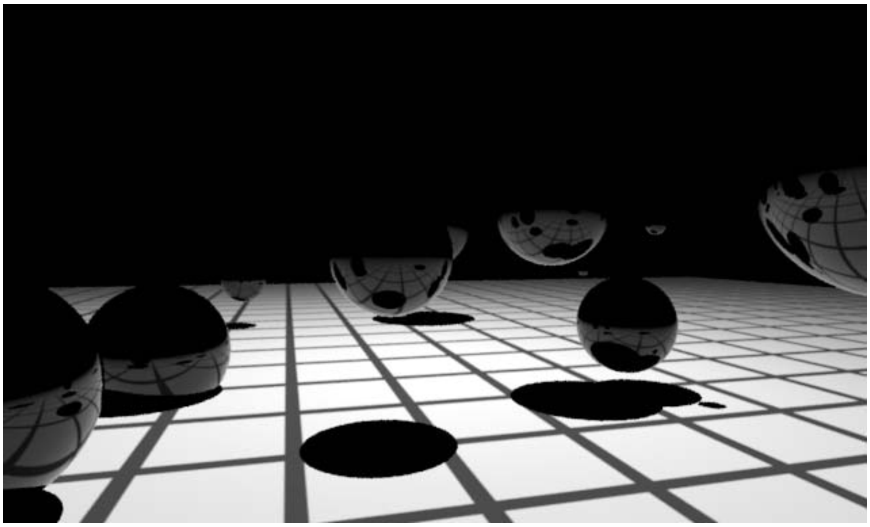
Spheres-over-plane.pbrt (mirror 10)



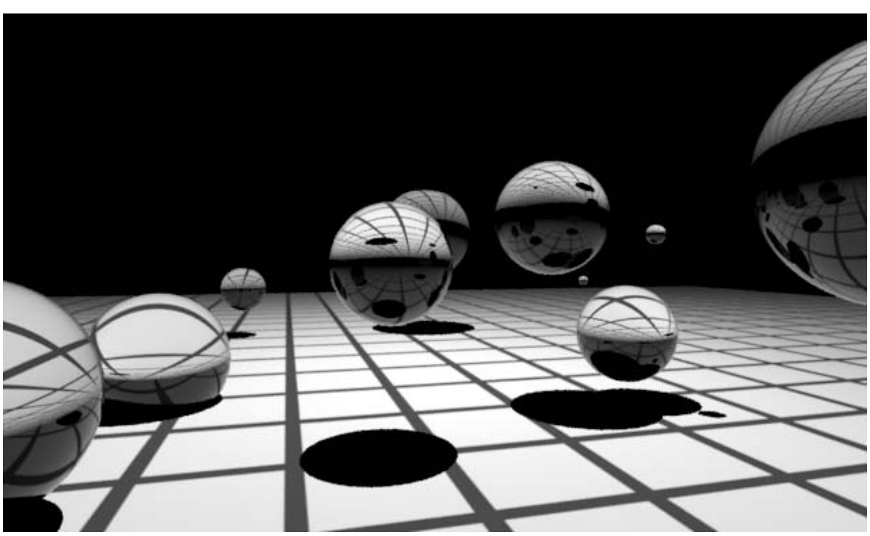
Spheres-over-plane.pbrt (glass 0)



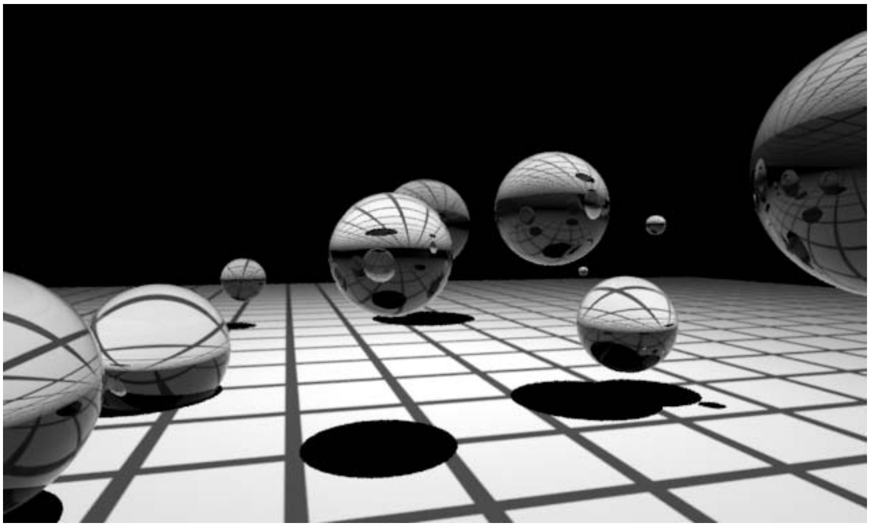
Spheres-over-plane.pbrt (glass 1)



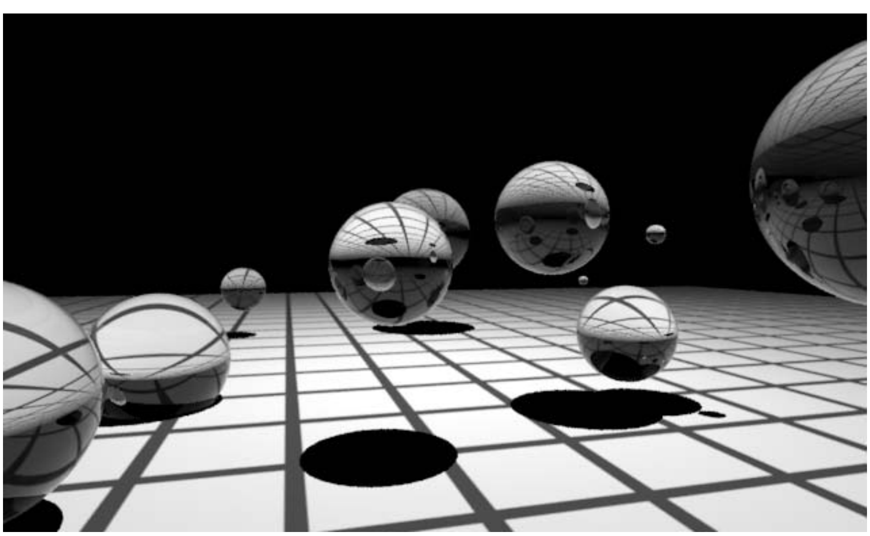
Spheres-over-plane.pbrt (glass 2)



Spheres-over-plane.pbrt (glass 5)



Spheres-over-plane.pbrt (glass 10)

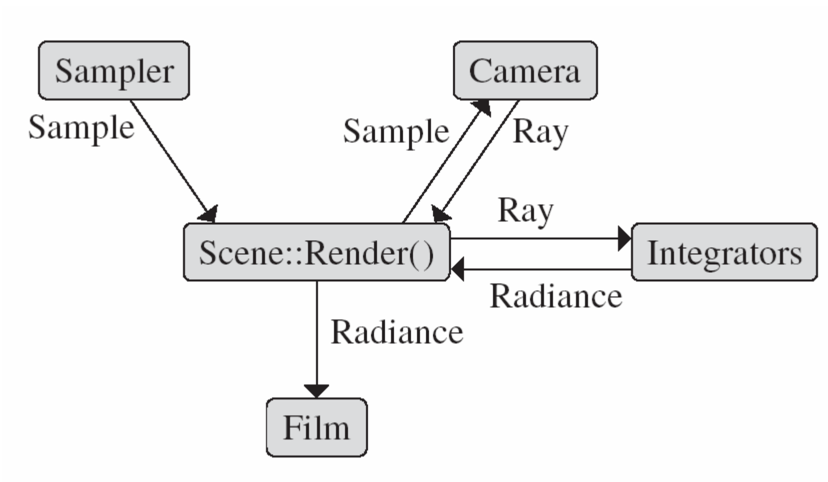


Spheres-over-plane.pbrt (glass 10)

Table 1.1: Plug-ins. pbrt supports 13 types of plug-in objects that can be loaded at run time based on the contents of the scene description file. The system can be extended with new plug-ins, without needing to be recompiled itself.

Base class	Directory	Section
Shape	shapes/	3.1
Primitive	accelerators/	4.1
Camera	cameras/	6.1
Film	film/	8.1
Filter	filters/	7.6
Sampler	samplers/	7.2
ToneMap	tonemaps/	8.4
Material	materials/	10.2
Texture	textures/	11.3
VolumeRegion	volumes/	12.3
Light	lights/	13.1
SurfaceIntegrator	integrators/	16
VolumeIntegrator	integrators/	17

PBRT Architecture



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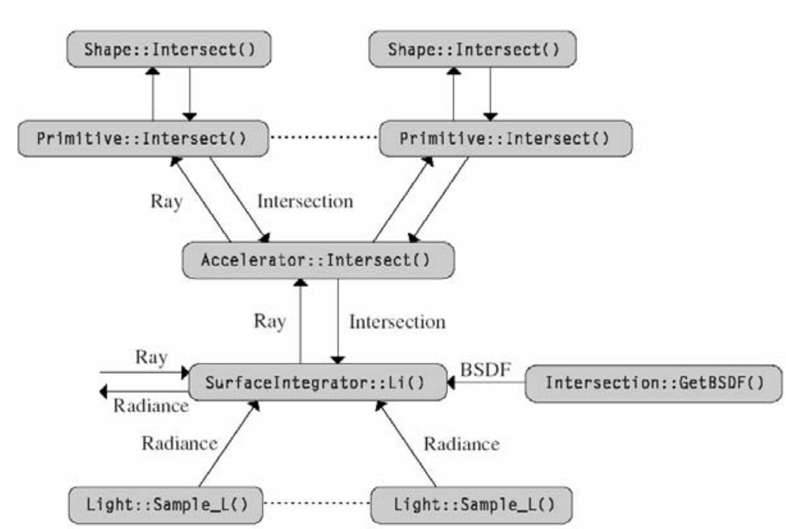
PBRT Architecture



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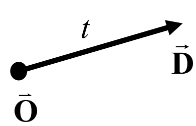
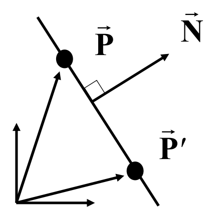
Ray-Surface Intersection

Ray-Plane Intersection

Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\,\vec{\mathbf{D}}$

$0 \le t < \infty$





Plane: $(\vec{\mathbf{P}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = 0$

$ax + by + cz + d = 0$

Solve for intersection

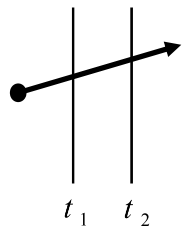
Substitute ray equation into plane equation

$$(\vec{\mathbf{P}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = (\vec{\mathbf{O}} + t\vec{\mathbf{D}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}} = 0$$

$$t = -\frac{(\vec{\mathbf{O}} - \vec{\mathbf{P}}') \bullet \vec{\mathbf{N}}}{\vec{\mathbf{D}} \bullet \vec{\mathbf{N}}}$$

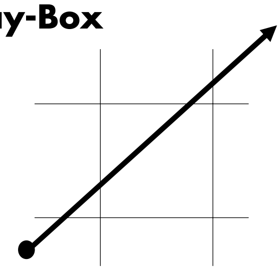
Ray-Polyhedra

Ray-Slab

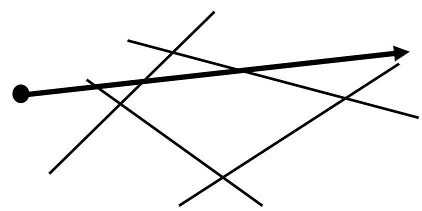


Note: Procedural geometry

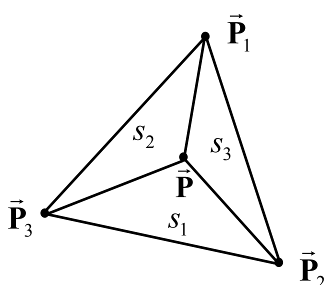
Ray-Box



Ray-Convex Polyhedra



Ray-Triangle Intersection 1



$$s_1 = \text{area}(\Delta \mathbf{P}\mathbf{P}_2\mathbf{P}_3)$$

$$s_2 = \text{area}(\Delta \mathbf{P}\mathbf{P}_3\mathbf{P}_1)$$

$$s_3 = \text{area}(\Delta \mathbf{P}\mathbf{P}_1\mathbf{P}_2)$$

Barycentric coordinates

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

Inside triangle criteria

$$0 \le s_1 \le 1$$

$$0 \le s_2 \le 1$$

$$0 \le s_3 \le 1$$

$$s_1 + s_2 + s_3 = 1$$

2D Homogeneous Coordinates

$$\vec{\mathbf{P}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{\mathbf{L}}_1 \times \vec{\mathbf{L}}_2 = \begin{bmatrix} b_1 c_2 - c_1 b_2 \\ c_1 a_2 - a_1 c_2 \\ a_1 b_2 - b_1 a_2 \end{bmatrix}$$

$$\vec{\mathbf{L}} = \begin{bmatrix} a \\ c \\ c \end{bmatrix} = \vec{\mathbf{P}}_1 \times \vec{\mathbf{P}}_2 = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ z_1 x_2 - x_1 z_2 \\ x_1 y_2 - y_1 x_2 \end{bmatrix}$$

$$\vec{\mathbf{L}} \bullet \vec{\mathbf{P}} = ax + by + cz$$

2D Homogeneous Coordinates

$$\vec{\mathbf{P}}_1 \bullet \vec{\mathbf{P}}_2 \times \vec{\mathbf{P}}_3 = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$\text{area}(\mathbf{P}_1 \mathbf{P}_2 \mathbf{P}_3) \propto \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

Ray-Triangle Intersection 2

$$\vec{\mathbf{P}} = s_1\vec{\mathbf{P}}_1 + s_2\vec{\mathbf{P}}_2 + s_3\vec{\mathbf{P}}_3$$

$$\begin{bmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{bmatrix}\begin{bmatrix}s_1\\ s_2\\ s_3\end{bmatrix} = \begin{bmatrix}\mathbf{P}\end{bmatrix}$$

$$\begin{bmatrix}s_1\\ s_2\\ s_3\end{bmatrix} = \begin{bmatrix}\mathbf{P}_2\times\mathbf{P}_3\\ \mathbf{P}_3\times\mathbf{P}_1\\ \mathbf{P}_1\times\mathbf{P}_2\end{bmatrix}\begin{bmatrix}\mathbf{P}\end{bmatrix}$$

Ray-Triangle Intersection 3

$$s_1 = \frac{\begin{vmatrix}\mathbf{P} & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}} = \mathbf{P}\bullet\frac{\mathbf{P}_2\times\mathbf{P}_3}{\Delta} = \mathbf{P}\bullet\mathbf{P}_2\times\mathbf{P}_3$$

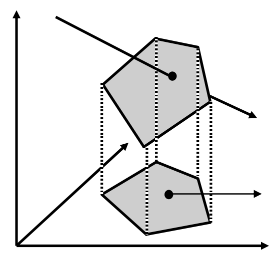
$$s_2 = \frac{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P} & \mathbf{P}_3\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}} = \mathbf{P}\bullet\frac{\mathbf{P}_3\times\mathbf{P}_1}{\Delta} = \mathbf{P}\bullet\mathbf{P}_3\times\mathbf{P}_1$$

$$s_3 = \frac{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}\end{vmatrix}}{\begin{vmatrix}\mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3\end{vmatrix}} = \mathbf{P}\bullet\frac{\mathbf{P}_1\times\mathbf{P}_2}{\Delta} = \mathbf{P}\bullet\mathbf{P}_1\times\mathbf{P}_2$$

$$\begin{bmatrix}s_1\\ s_2\\ s_3\end{bmatrix} = \begin{bmatrix}\mathbf{P}_2\times\mathbf{P}_3\\ \mathbf{P}_3\times\mathbf{P}_1\\ \mathbf{P}_1\times\mathbf{P}_2\end{bmatrix}\begin{bmatrix}\mathbf{P}\end{bmatrix}$$

Ray-Polygon Intersection

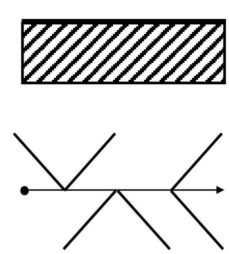
1. Find intersection with plane of support
2. Test whether point is inside 3D polygon
   a. Project onto *xy* plane
   b. Test whether point is inside 2D polygon



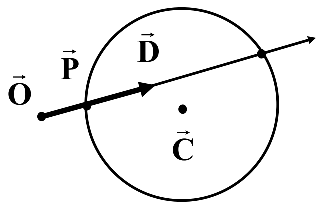
Point in Polygon

```
inside(vert v[], int n, float x, float y)
{
  int cross=0; float x0, y0, x1, y1;

  x0 = v[n-1].x - x;
  y0 = v[n-1].y - y;
  while( n-- ) {
    x1 = v->x - x;
    y1 = v->y - y;
    if( y0 > 0 ) {
      if( y1 <= 0 )
        if( x1*y0 > y1*x0 ) cross++;
    }
    else {
      if( y1 > 0 )
        if( x0*y1 > y0*x1 ) cross++;
    }
    x0 = x1; y0 = y1; v++;
  }
  return cross & 1;
}
```



Ray-Sphere Intersection



Ray: $\vec{\mathbf{P}} = \vec{\mathbf{O}} + t\vec{\mathbf{D}}$

Sphere: $\left(\vec{\mathbf{P}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$

$$\left(\vec{\mathbf{O}} + t\,\vec{\mathbf{D}} - \vec{\mathbf{C}}\right)^2 - R^2 = 0$$

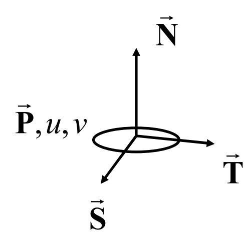
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$at^2 + bt + c = 0$$
$$a = \vec{\mathbf{D}}^2$$
$$b = 2(\vec{\mathbf{O}} - \vec{\mathbf{C}}) \bullet \vec{\mathbf{D}}$$
$$c = (\vec{\mathbf{O}} - \vec{\mathbf{C}})^2 - R^2$$

Geometric Methods

Methods

- **Find normal and tangents**
- **Find surface parameters**



e.g. Sphere

Normal $\vec{\mathbf{N}} = \vec{\mathbf{P}} - \vec{\mathbf{C}}$

Parameters

$x = \sin\theta\cos\phi$ $\qquad \phi = \tan^{-1}(x, y)$

$y = \sin\theta\sin\phi$ $\qquad \theta = \cos^{-1} z$

$z = \cos\theta$

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Ray-Implicit Surface Intersection

$$f(x,y,z)=0$$

$$x = x_0 + x_1\,t$$
$$y = y_0 + y_1\,t$$
$$z = z_0 + z_1\,t$$

$$f^*(t)=0$$

1. Substitute ray equation
2. Find *positive, real* roots

Univariate root finding

- Newton's method
- *Regula-falsi*
- Interval methods
- Heuristics

Ray-Algebraic Surface Intersection

$$p_n(x,y,z)=0$$

$$x = x_0 + x_1\,t$$
$$y = y_0 + y_1\,t$$
$$z = z_0 + z_1\,t$$

$$p_n^*(t)=0$$

Degree *n*

Linear: Plane

Quadric: Spheres, …

Quartic: Tori

Polynomial root finding

- Quadratic, cubic, quartic
- Bezier/Bernoulli basis
- Descartes' rule of signs
- Sturm sequences

History

Polygons	**Appel '68**
Quadrics, CSG	**Goldstein & Nagel '71**
Tori	**Roth '82**
Bicubic patches	**Whitted '80, Kajiya '82**
Superquadrics	**Edwards & Barr '83**
Algebraic surfaces	**Hanrahan '82**
Swept surfaces	**Kajiya '83, van Wijk '84**
Fractals	**Kajiya '83**
Height fields	**Coquillart & Gangnet '84, Musgrave '88**
Deformations	**Barr '86**
Subdivision surfs.	**Kobbelt, Daubert, Siedel, '98**

P. Hanrahan, A survey of ray-surface intersection algorithms