

Reflection Models I

Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture

- Glossy and specular reflection models
- Rough surfaces and microfacets
- Self-shadowing
- Anisotropic reflection models

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Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

- Spectra and Color [Moon Spectra]
- Polarization
- Directional distribution

Theories

- Phenomenological
- Physical

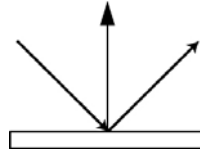
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Types of Reflection Functions

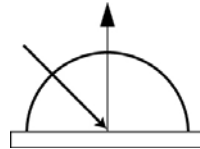
Ideal Specular

- Reflection Law
- Mirror



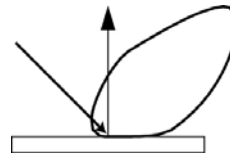
Ideal Diffuse

- Lambert's Law
- Matte



Specular

- Glossy
- Directional diffuse



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Materials



Plastic



Metal



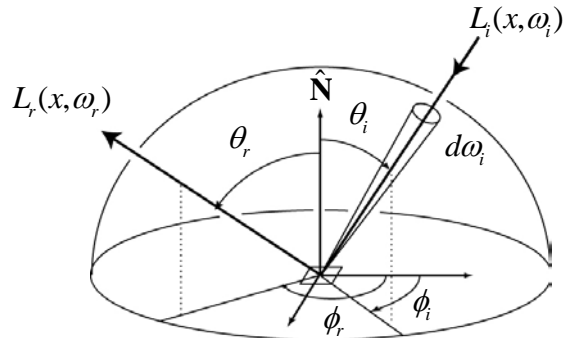
Matte

From Apodaca and Gritz, *Advanced RenderMan*

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The Reflection Equation



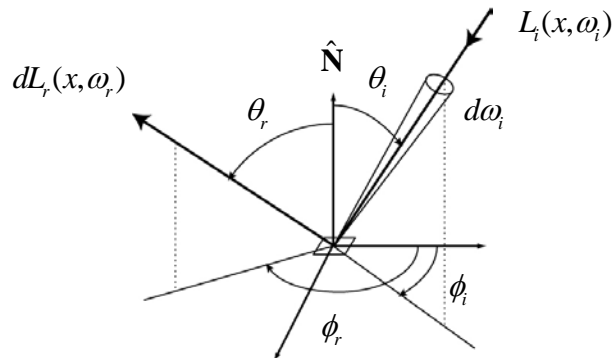
$$L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

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The BRDF

Bidirectional Reflectance-Distribution Function



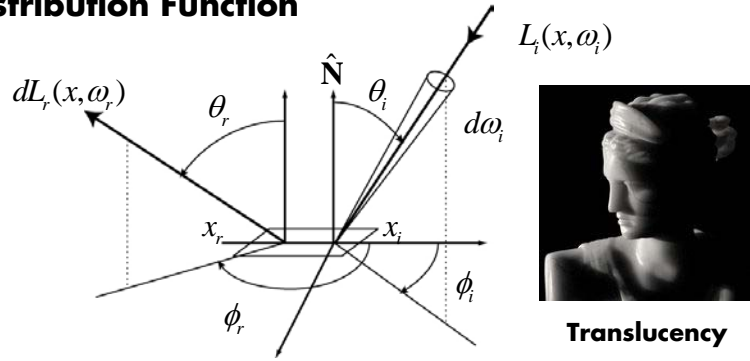
$$f_r(\omega_i \rightarrow \omega_r) \equiv \frac{dL_r(\omega_i \rightarrow \omega_r)}{dE_i} \left[\frac{1}{sr} \right]$$

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The BSSRDF

Bidirectional Surface Scattering Reflectance-Distribution Function

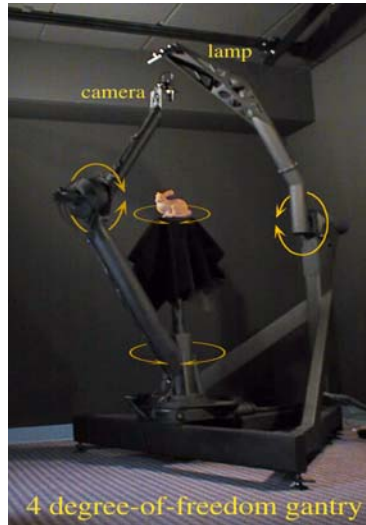
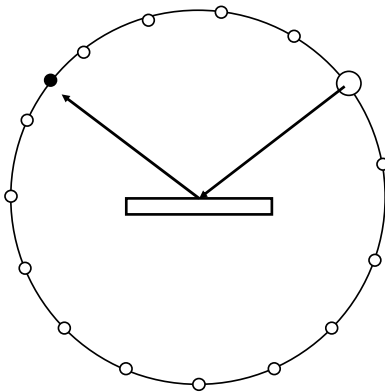


$$S(x_i, \omega_i \rightarrow x_r, \omega_r) \equiv \frac{dL_r(x_r, \omega_r \rightarrow x_i, \omega_i)}{d\Phi_i}$$

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Gonioreflectometer

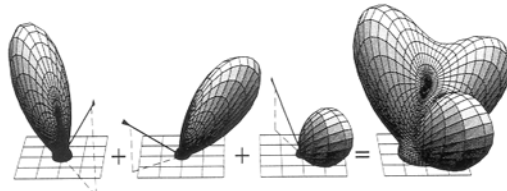


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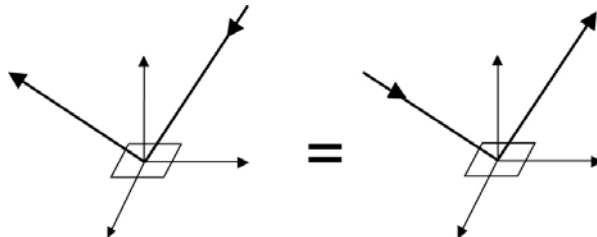
Properties of BRDF's

1. Linear



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle $f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r)$



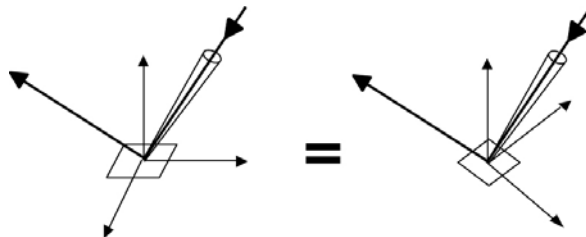
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Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i)$$



Reciprocity and isotropy

$$f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|)$$

4. Energy conservation

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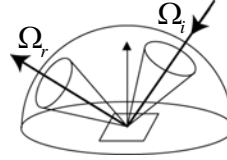
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Energy Conservation

$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r) \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i}$$

$$= \frac{\int \int f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i}$$

$$\leq 1$$



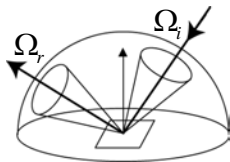
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The Reflectance

Definition: Reflectance is ratio of reflected to incident power

$$\rho(\Omega_i \rightarrow \Omega_r) \equiv \frac{\int \int f_r(\omega_i \rightarrow \omega_r) \cos \theta_i d\omega_i \cos \theta_r d\omega_r}{\int_{\Omega_i} \cos \theta_i d\omega_i}$$



$$\leq \frac{\int \int f_r(\omega_i \rightarrow \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i \cos \theta_r d\omega_r}{\int_{\Omega_i} L_i(\omega_i) \cos \theta_i d\omega_i}$$

Conservation of energy: $0 < \rho < 1$

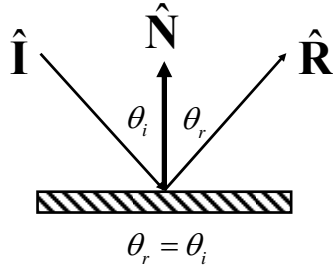
3 by 3 set of possibilities: $\{d\omega_i, \Omega_i, H_i^2\} \times \{d\omega_r, \Omega_r, H_r^2\}$

Units: ρ [dimensionless], f_r [1/steradians]

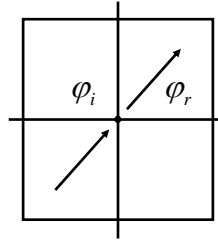
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Law of Reflection



$$\theta_r = \theta_i$$



$$\varphi_r = (\varphi_i + \pi) \bmod 2\pi$$

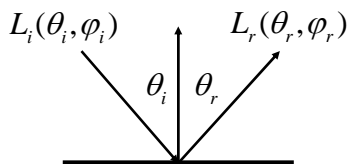
$$\hat{\mathbf{R}} + (-\hat{\mathbf{I}}) = 2 \cos \theta \hat{\mathbf{N}} = -2(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})\hat{\mathbf{N}}$$

$$\hat{\mathbf{R}} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \cdot \hat{\mathbf{N}})\hat{\mathbf{N}}$$

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Ideal Reflection (Mirror)



$$L_{r,m}(\theta_r, \varphi_r) = L_i(\theta_r, \varphi_r \pm \pi)$$

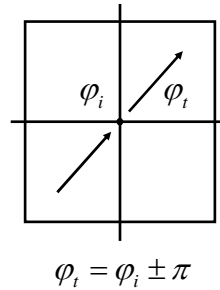
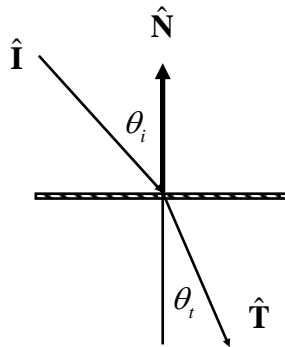
$$f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) = \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi)$$

$$\begin{aligned} L_{r,m}(\theta_r, \varphi_r) &= \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i) \cos \theta_i d \cos \theta_i d \varphi_i \\ &= \int \frac{\delta(\cos \theta_i - \cos \theta_r)}{\cos \theta_i} \delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i) \cos \theta_i d \cos \theta_i d \varphi_i \\ &= L_i(\theta_r, \varphi_r \pm \pi) \end{aligned}$$

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Snell's Law



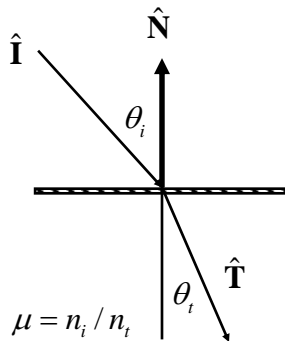
$$n_i \sin \theta_i = n_t \sin \theta_t$$

$$n_i \hat{N} \times \hat{I} = n_t \hat{N} \times \hat{T}$$

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Law of Refraction



$$\hat{N} \times \hat{T} = \mu \hat{N} \times \hat{I}$$

$$\hat{N} \times (\hat{T} - \mu \hat{I}) = 0$$

$$\hat{T} = \mu \hat{I} + \gamma \hat{N}$$

$$\hat{T}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \hat{I} \cdot \hat{N}$$

$$\gamma = -\mu \hat{I} \cdot \hat{N} \pm \left\{ 1 - \mu^2 \left(1 - (\hat{I} \cdot \hat{N})^2 \right) \right\}^{1/2}$$

$$= \mu \cos \theta_i \pm \left\{ 1 - \mu^2 \sin^2 \theta_i \right\}^{1/2}$$

$$= \mu \cos \theta_i \pm \cos \theta_t \quad \leftarrow \gamma = \mu - 1$$

$$= \mu \cos \theta_i - \cos \theta_t$$

Total internal reflection:

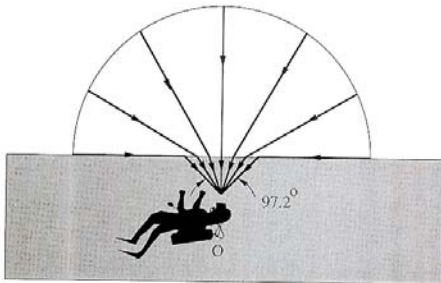
$$1 - \mu^2 (1 - (\hat{I} \cdot \hat{N})^2) < 0$$

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Optical Manhole

Total internal reflection



$$n_w = \frac{4}{3}$$



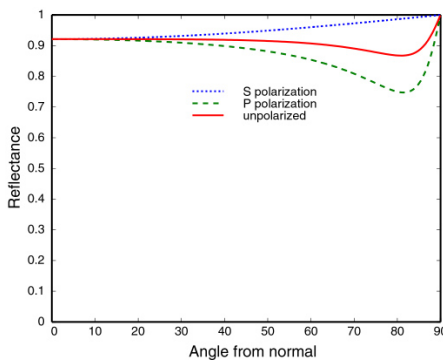
From Livingston and Lynch

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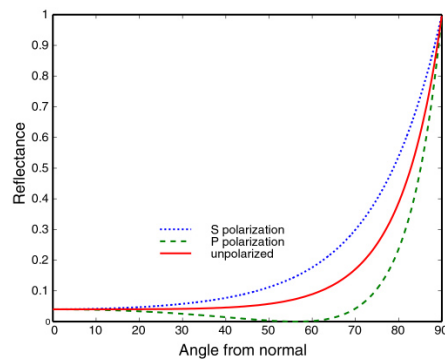
Fresnel Reflectance

Metal (Aluminum)



Gold $F(0)=0.82$
Silver $F(0)=0.95$

Dielectric (N=1.5)



Glass $n=1.5$ $F(0)=0.04$
Diamond $n=2.4$ $F(0)=0.15$

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

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Experiment

Reflections from a shiny floor



From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

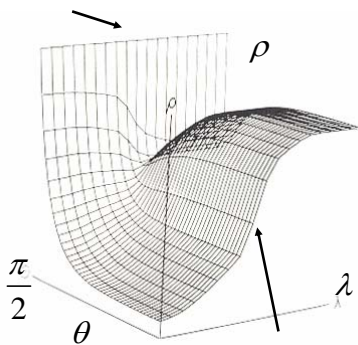
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Cook-Torrance Model for Metals

Reflectance of Copper as a function of wavelength and angle of incidence

Light spectra



Copper spectra



Measured Reflectance



Approximated Reflectance

Cook-Torrance approximation

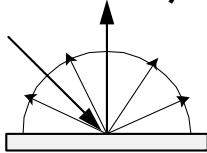
$$R = R(0) + R(\pi/2) \left[\frac{F(\theta) - F(0)}{F(\pi/2) - F(0)} \right]$$

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Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$f_{r,d} = c$$

$$\begin{aligned} L_{r,d}(\omega_r) &= \int f_{r,d} L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,d} \int L_i(\omega_i) \cos \theta_i d\omega_i \\ &= f_{r,d} E \end{aligned}$$

$$M = \int L_r(\omega_r) \cos \theta_r d\omega_r = L_r \int \cos \theta_r d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \Rightarrow f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos \theta_s$

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"Diffuse" Reflection

Theoretical

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

Experimental

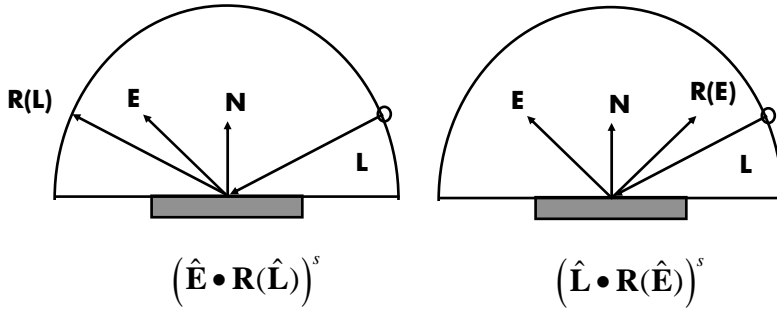
- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufacturers attempt to create ideal diffuse

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Phong Model



Reciprocity: $(\hat{E} \cdot \mathbf{R}(\hat{L}))^s = (\hat{L} \cdot \mathbf{R}(\hat{E}))^s$

Distributed light source!

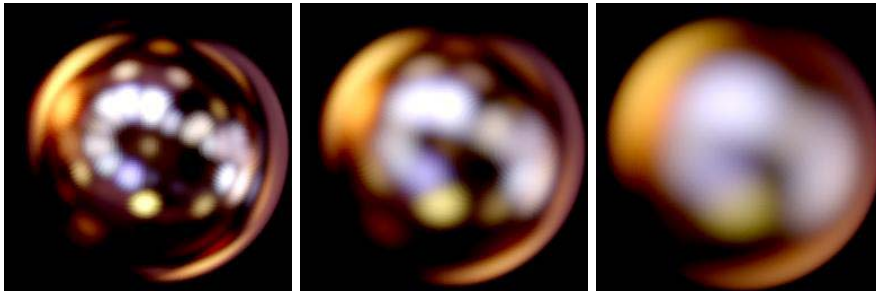
Phong Model



Mirror



Diffuse



Properties of the Phong Model

Energy normalize Phong Model

$$\begin{aligned}\rho(H^2 \rightarrow \omega_r) &= \int_{H^2(\hat{\mathbf{N}})} (\hat{\mathbf{L}} \cdot \mathbf{R}(\hat{\mathbf{E}}))^s \cos \theta_i d\omega_i \\ &\leq \int_{H^2(\hat{\mathbf{R}})} (\hat{\mathbf{L}} \cdot \mathbf{R}(\hat{\mathbf{E}}))^s d\omega_{ir} \\ &\leq \int_{H^2} \cos^s \theta d\omega = \frac{2\pi}{s+1}\end{aligned}$$