Reflection Models

Last lecture

- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

Today

- Phong and microfacet models
- Gaussian height field on surface
- Self-shadowing
- Torrance-Sparrow model

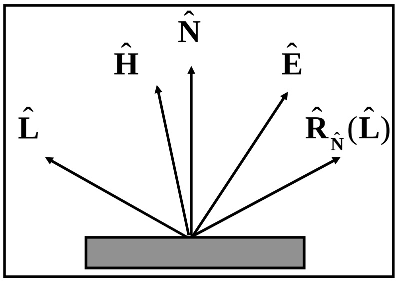
Next

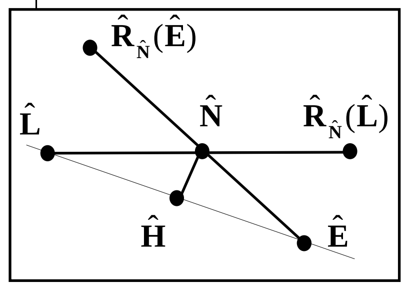
- Anisotropic: Grooves and hair
- Translucency: Skin

Phong Model

Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$$





$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

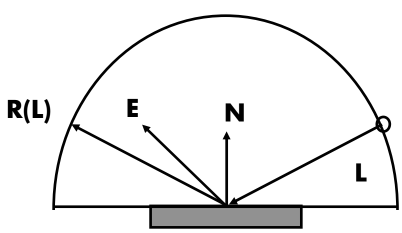
$$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$$

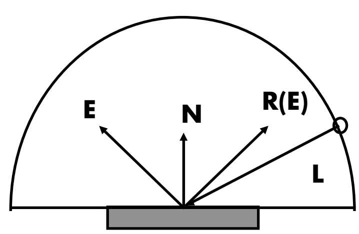
$$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{N}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{N}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$$

$$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$$

$$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$$

Phong Model



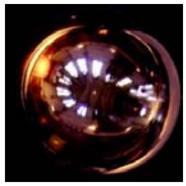


$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{N}}(\hat{\mathbf{L}})\right)^s \qquad \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{N}}(\hat{\mathbf{E}})\right)^s$$

Reciprocity: $\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

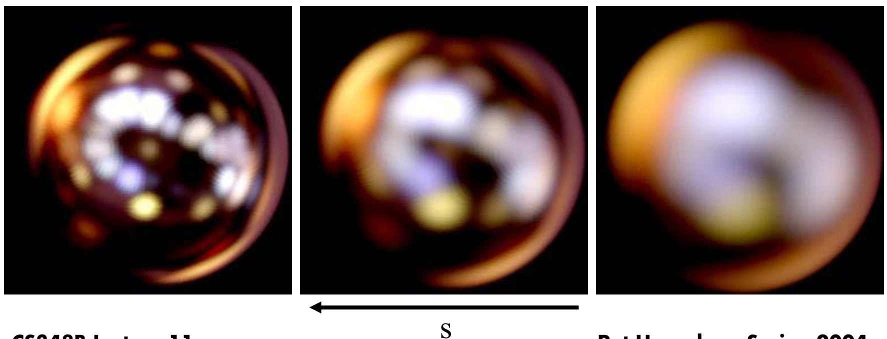
Distributed light source!

Phong Model



Mirror

Diffuse



Energy normalization

Energy normalize Phong Model

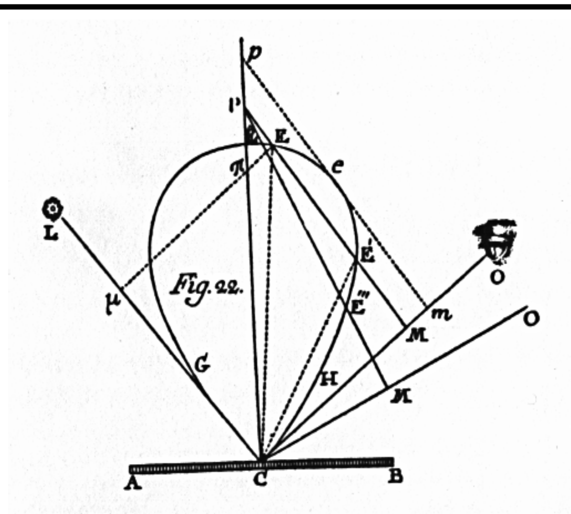
$$\rho(H^2 \to \omega_r) = \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s \cos\theta_i \, d\omega_i$$

$$\leq \int_{H^2(\hat{\mathbf{R}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_{ir}$$

$$\leq \int_{H^2} \cos^s\theta \, d\omega = \frac{2\pi}{s+1}$$

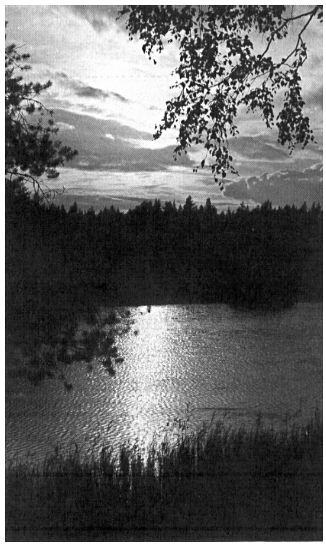
Microfacet Model

Bouguer's "little faces"

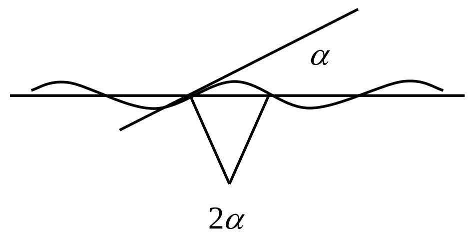


P. Bouguer, *Treatise on Optics*, 1760

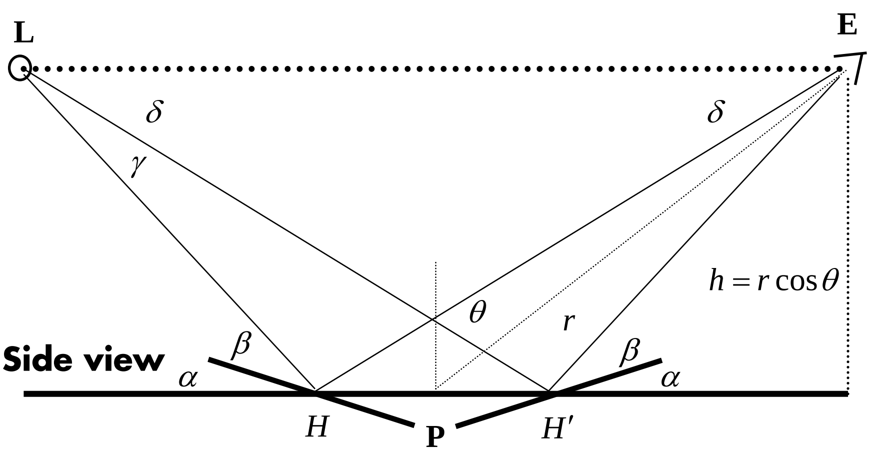
Reflection of the Sun from the Sea



***Minnaert**, **Light and Color in the Outdoors**, p. 28*



Reflection Angles

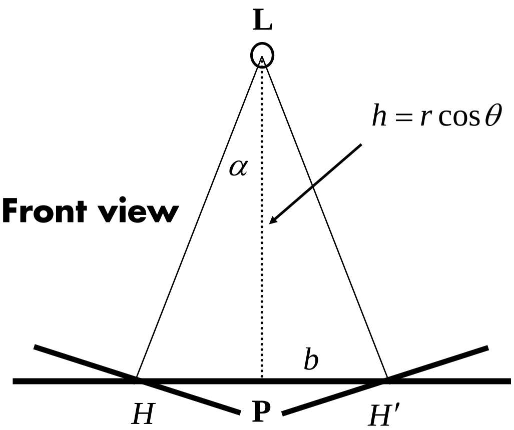


Assume L and E are at the same height h

$$\alpha + \beta = \gamma + \delta$$
$$\beta - \alpha = \delta$$
$$\Rightarrow \quad \gamma = 2\alpha$$

Reflection Angles

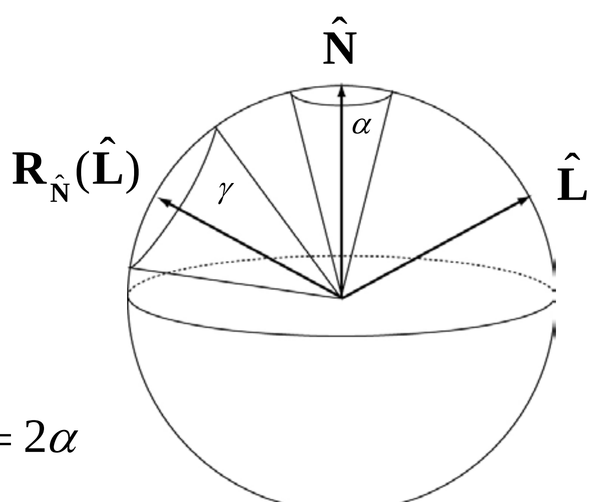
Front view



$$\tan\alpha = \frac{b}{h}$$

$$\tan\psi = \frac{b}{r} = \frac{h}{r}\tan\alpha = \tan\alpha\cos\theta$$

Analysis on the Sphere



$$\gamma = 2\alpha$$

$$\tan\psi = \tan\alpha\cos\theta$$

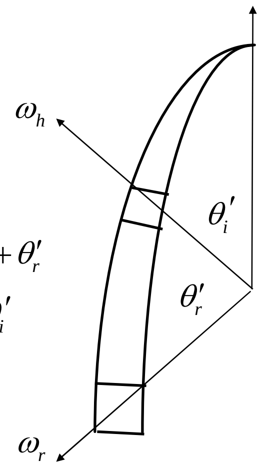
Solid Angle Distributions

$\theta_r' = \theta_i'$

$\theta_r = \theta_i' + \theta_r'$

$\quad = 2\theta_i'$

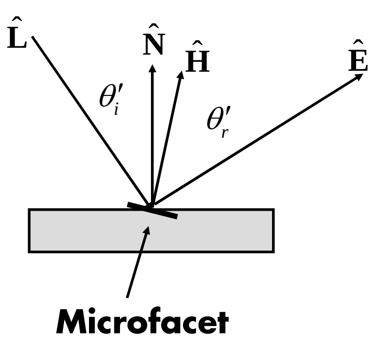
$\varphi_r = \varphi_i$



$$
\begin{aligned}
d\omega_r &= \sin\theta_r \, d\theta_r \, d\varphi_r \\
&= \left(\sin 2\theta_i'\right) 2 d\theta_i' \, d\varphi_i \\
&= \left(2\sin\theta_i' \cos\theta_i'\right) 2 d\theta_i' \, d\varphi_i \\
&= 4\cos\theta_i' \sin\theta_i' d\theta_i' \, d\varphi_i \\
&= 4\cos\theta_i' \, d\omega_h
\end{aligned}
$$

$$\frac{d\omega_h}{d\omega_r} = \frac{1}{4\cos\theta_i'}$$

Microfacet Distributions



Total projected area

$$\int_{H^2} dA(\omega_h) \cos\theta_h \, d\omega_h = dA$$

Probability distribution

$$\int_{H^2} D(\omega_h) \cos\theta_h \, d\omega_h = 1$$

Area distribution $dA(\omega_h)$

Microfacet distribution $D(\omega_h) \equiv dA(\omega_h) / dA$

Microfacet Distribution Functions

Isotropic distributions $D(\omega_h) \Rightarrow D(\alpha)$

Characterize by half-angle β $\quad D(\beta) = \frac{1}{2}$

Examples:

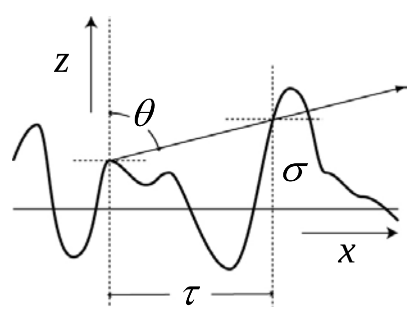
- **Blinn** $\quad D_1(\alpha) = \cos^{c_1}\alpha \qquad c_1 = \frac{\ln 2}{\ln\cos\beta}$
- **Torrance-Sparrow** $\quad D_2(\alpha) = e^{-(c_2\alpha)^2} \qquad c_2 = \frac{\sqrt{2}}{\beta}$
- **Trowbridge-Reitz** $\quad D_3(\alpha) = \frac{c_3^2}{(1-c_3^2)\cos^2\alpha - 1}$

$$c_3 = \left(\frac{\cos^2\beta - 1}{\cos^2\beta - \sqrt{2}}\right)^{1/2}$$

Gaussian Rough Surface

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



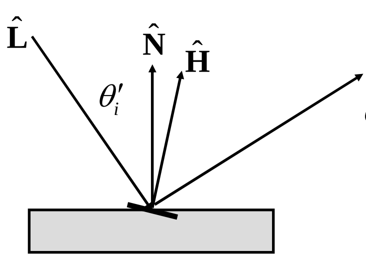
Gaussian distribution of slopes

$$D(\alpha) = \frac{1}{\sqrt{\pi}m^2\cos^2\alpha} e^{-\frac{\tan^2\alpha}{m^2}} \qquad m = \frac{2\sigma}{\tau}$$

Beckmann

Torrance-Sparrow Model

Torrance-Sparrow Model



$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$
$$\cos\theta_i' = \hat{\mathbf{L}} \bullet \hat{\mathbf{H}}$$

$$\begin{aligned} d\Phi_h &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, dA(\omega_h) d\omega_h \\ &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, dA d\omega_h \end{aligned}$$

$$dA(\omega_h) = D(\omega_h)\, dA$$

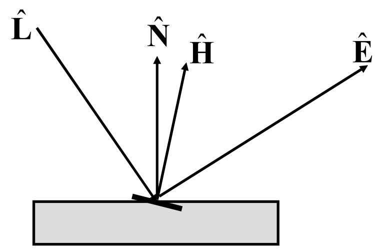
$$d\Phi_r = dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$

$$d\Phi_r = d\Phi_h$$

$$\begin{aligned} \therefore\, & dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA \\ & = L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA \end{aligned}$$

Torrance-Sparrow Model

$$dL_r(\omega_i \to \omega_r)\cos\theta_r \, d\omega_r \, dA$$
$$= L_i(\omega_i)\cos\theta_i' \, d\omega_i' \, D(\omega_h)\, d\omega_h \, dA$$

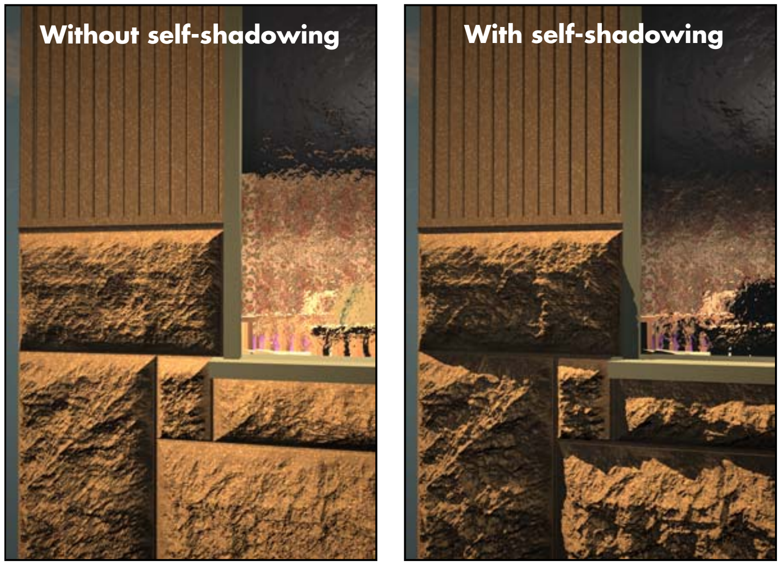


$$d\omega_i' = d\omega_i$$

$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i(\omega_i)}$$
$$= \frac{L_i(\omega_i)\cos\theta_i' \, d\omega_i' \, D(\omega_h)\, d\omega_h \, dA}{\left(\cos\theta_r d\omega_r dA\right)\left(L_i(\omega_i)\cos\theta_i \, d\omega_i\right)}$$
$$= \frac{D(\omega_h)}{\cos\theta_i \cos\theta_r} \cos\theta_i' \frac{d\omega_h}{d\omega_r}$$
$$= \frac{D(\omega_h)}{4\cos\theta_i \cos\theta_r}$$

Self-Shadowing

Shadows on Rough Surfaces



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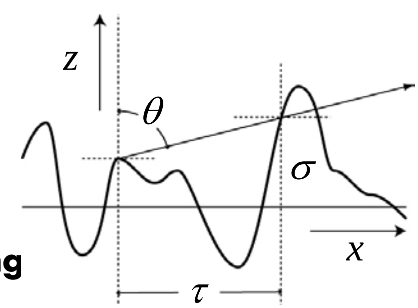
Self-Shadowing Function



Probability of shadowing

$$S(\theta) = \frac{\left[1 - \frac{1}{2}\operatorname{erfc}\left(\mu/\sqrt{2}m\right)\right]}{1 + \Lambda(\mu)}$$

$$2\Lambda(\mu) = \left(\sqrt{\frac{2}{\pi}}\right)\frac{m}{\mu}e^{-\mu^2/2m^2} - \operatorname{erfc}\left(\mu/\sqrt{2}m\right)$$

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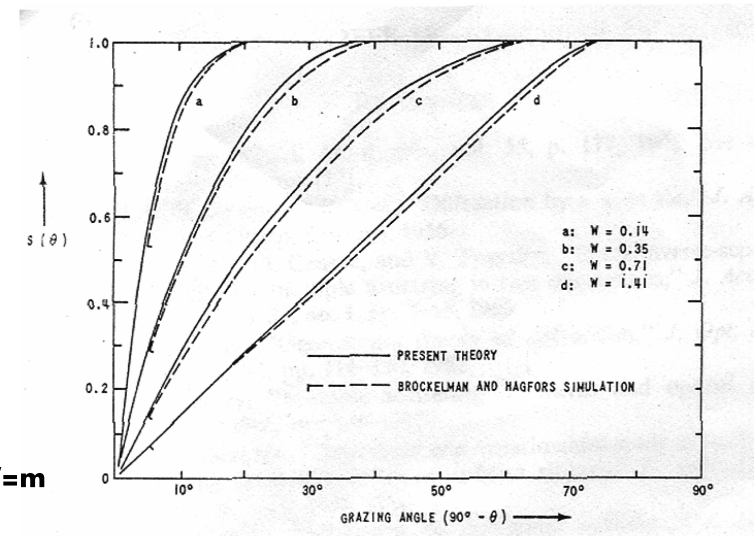
Self-Shadowing Function



W=m

From Smith, 1967

Self-Consistency Condition

$$\int S(\theta) D(\alpha) \cos\theta' \, d\omega_{\alpha} = \cos\theta$$

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

Torrance-Sparrow Theory

$$f_r(\omega_i \to \omega_r) = \frac{F(\theta_i')S(\theta_i)S(\theta_r)D(\alpha)}{4\cos\theta_i \cos\theta_r}$$

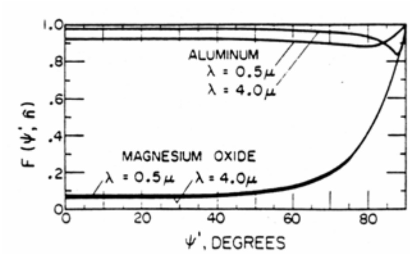


FIG. 6. Fresnel reflectance.

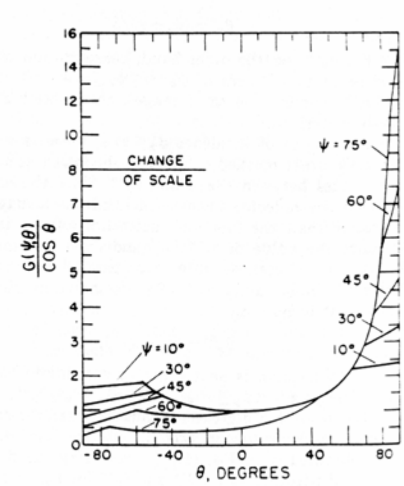
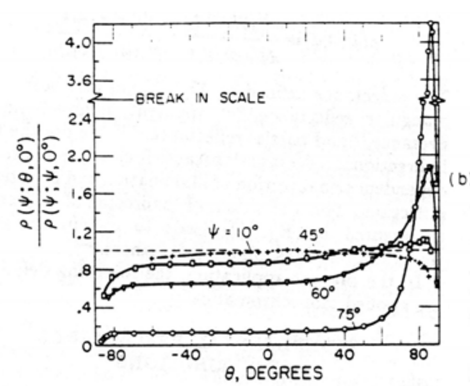
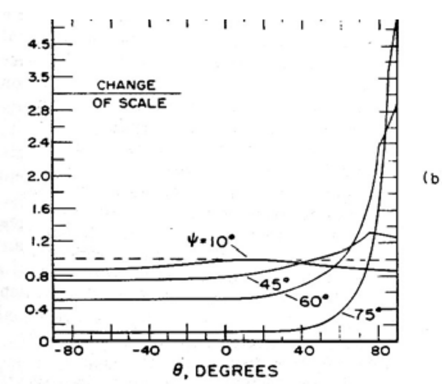


FIG. 7. The factor $G(\psi,\theta)/\cos\theta$ in the plane of incidence for various incidence angles ψ.

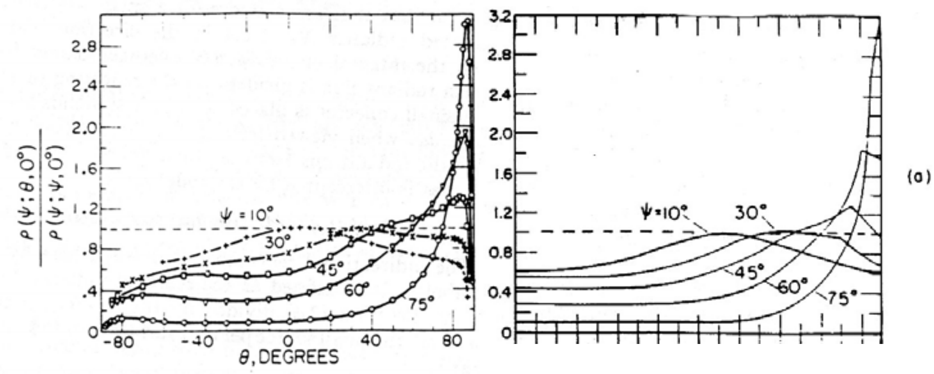
Torrance-Sparrow Comparison





Magnesium Oxide

Torrance-Sparrow Comparison



Aluminum

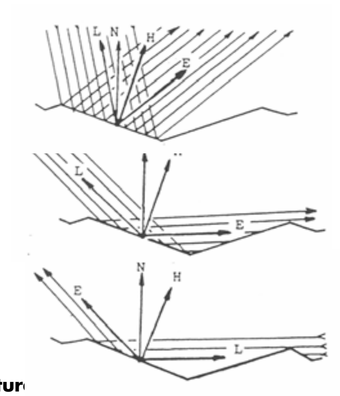
Self-Shadowing

V-Groove Model

Self-Shadowing: V-Groove Model

Assumptions (Torrance-Sparrow)

1. **Symmetric, longitudinal, isotropically-distributed**
2. **Upper edges lie in plane**



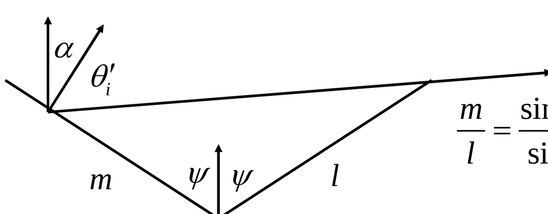
$$G = \min(G_a, G_b, G_c)$$

$$G_a = 1$$

$$G_b = \frac{2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}})}{(\hat{\mathbf{H}} \bullet \hat{\mathbf{E}})}$$

$$G_c = \frac{2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{L}})}{(\hat{\mathbf{H}} \bullet \hat{\mathbf{L}})}$$

Self-Shadowing: V-Groove Model



$$\sin l = \cos\theta_i'$$
$$\cos l = \sin\theta_i'$$
$$\sin\psi = \cos\alpha$$
$$\cos\psi = \sin\alpha$$

$$\frac{m}{l} = \frac{\sin m}{\sin l}$$

$$\begin{aligned}
\sin m &= \sin l + 2\psi \\
&= \sin l \cos 2\psi + \cos l \sin 2\psi \\
&= \cos\theta_i' \cos 2\psi + \sin\theta_i' \sin 2\psi \\
&= \cos\theta_i'(1 - 2\sin^2\psi) + \sin\theta_i' 2\cos\psi\sin\psi \\
&= \cos\theta_i'(1 - 2\cos^2\alpha) + \sin\theta_i' 2\cos\alpha\sin\alpha \\
&= \cos\theta_i' - 2\cos\alpha\left(\cos\alpha\cos\theta_i' - \sin\alpha\sin\theta_i''\right) \\
&= \cos\theta_i' - 2\cos\alpha\cos\left(\alpha + \theta_i'\right) \\
&= \cos\theta_i' - 2\cos\alpha\cos\theta_r \\
&= \hat{\mathbf{H}} \bullet \hat{\mathbf{E}} - 2\left(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}}\right)\left(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}}\right)
\end{aligned}$$

$$\begin{aligned}
G &= 1 - \frac{m}{l} \\
&= 1 - \frac{\sin m}{\sin l} \\
&= \frac{\hat{\mathbf{H}} \bullet \hat{\mathbf{E}} - \hat{\mathbf{H}} \bullet \hat{\mathbf{E}} + 2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}})}{\hat{\mathbf{H}} \bullet \hat{\mathbf{E}}} \\
&= \frac{2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}})}{\hat{\mathbf{H}} \bullet \hat{\mathbf{E}}}
\end{aligned}$$