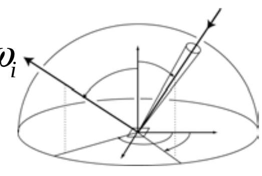
Illumination Models

To evaluate the reflection equation the illumination must be specified or computed.

$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \to \omega_r) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$



Direct (*local*) illumination

- **Light directly from light sources**
- **No shadows**

Indirect (*global*) illumination

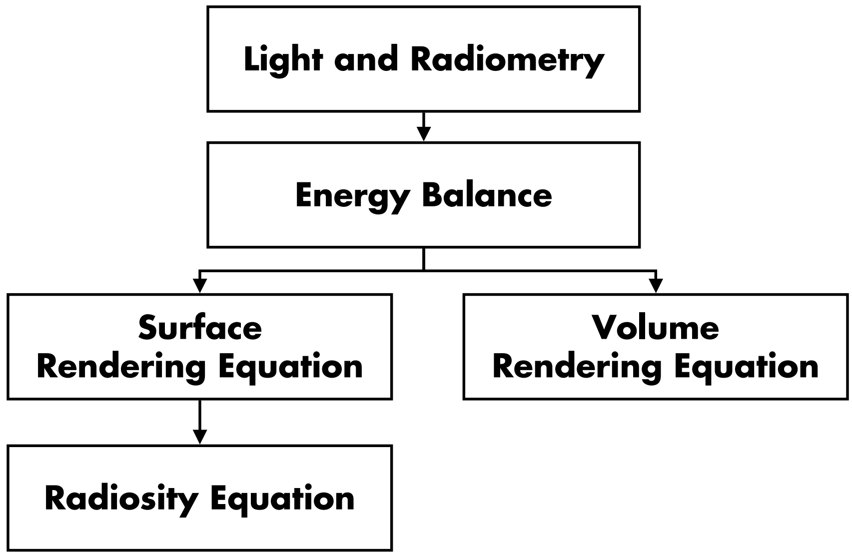
- **Shadows due to blocking light**
- **Interreflections**
- **Complete accounting of light energy**

To The Rendering Equation

Questions

1. How is light measured?
2. How is the spatial distribution of light energy described?
3. How is reflection from a surface characterized?
4. What are the conditions for equilibrium flow of light in an environment?

The Grand Scheme



Balance Equation

Accountability

[*outgoing*] - [*incoming*] = [*emitted*] - [*absorbed*]

- **Macro level**

  The total light energy put into the system must equal the energy leaving the system (usually, via heat).

$$\Phi_o - \Phi_i = \Phi_e - \Phi_a$$

- **Micro level**

  The energy flowing into a small region of phase space must equal the energy flowing out.

$$B(x) - E(x) = B_e(x) - E_a(x)$$

$$L_o(x,\omega) - L_i(x,\omega) = L_e(x,\omega) - L_a(x,\omega)$$

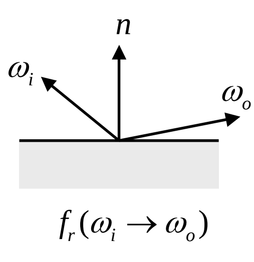
CS348B Lecture 14 Pat Hanrahan, Spring 2002

Surface Balance Equation

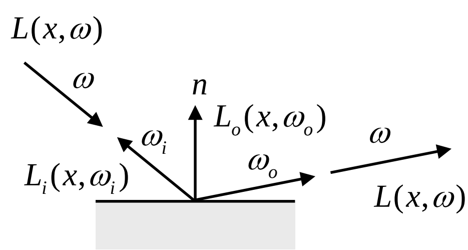
[*outgoing*] = [*emitted*] + [*reflected*]

$$L_o(x,\omega_o) = L_e(x,\omega_o) + L_r(x,\omega_o)$$
$$= L_e(x,\omega_o) + \int_{H^2} f_r(x,\omega_i \to \omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

Direction Conventions



BRDF



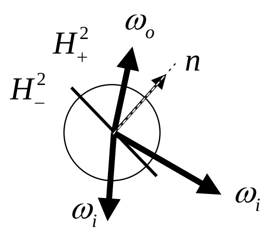
$$L_i(x,\omega_i) = L(x,-\omega_i)$$
$$L(x,\omega) = L_o(x,\omega_o)$$

Surface vs. Field Radiance

Surface Balance Equation

[*outgoing*] = [*emitted*] + [*reflected*] + [*transmitted*]

$$L_o(x,\omega_o) = L_e(x,\omega_o) + L_r(x,\omega_o) + L_t(x,\omega_o)$$



$$L_r(x,\omega_o) = \int_{H_+^2} f_r(x,\omega_i \to \omega_o) L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

$$L_t(x,\omega_o) = \int_{H_-^2} f_t(x,\omega_i \to \omega_o) L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

BTDF

$H_+^2(n) \quad \omega_i \bullet n(x) > 0$

$H_-^2(n) \quad \omega_i \bullet n(x) < 0$

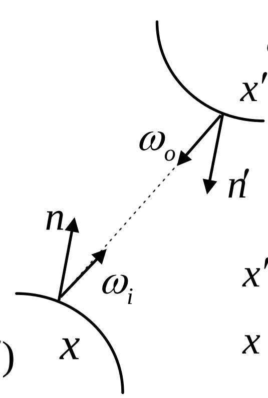
Two-Point Geometry

$$\omega(x,x') = \omega(x \to x') = \frac{x'-x}{|x'-x|}$$

Ray Tracing

$x^*(x,\omega)$

$\omega_o = \omega(x',x)$



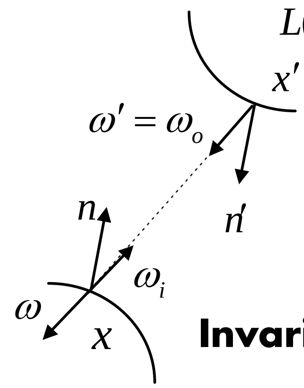
$x' = x^*(x,\omega_i)$

$x = x^*(x',\omega_o)$

$\omega_i = \omega(x,x')$

Coupling Equations

$$L(x',\omega') = L_o(x',\omega_o)$$



Invariance of radiance

$$L_i(x,\omega_i) = L(x,-\omega_i)$$

$$L(x,\omega) = L(x',\omega')$$

The Rendering Equation

Directional form

$$L(x,\omega) = L_e(x,\omega) + \int_{H^2} f_r(x,\omega' \to \omega)\, L(x^*(x,\omega'),-\omega') \cos\theta' \, d\omega'$$

Integrate over hemisphere of directions

Transport operator i.e. ray tracing

The Rendering Equation

Surface form

$$L(x,\omega) = L_e(x,\omega) + \int_{M^2} f_r(x,\omega(x,x') \to \omega)\, L(x',\omega(x',x))\, G(x,x')\, dA'(x')$$

Geometry term

Integrate over all surfaces

$$G(x,x') = \frac{\cos\theta_i \cos\theta_o'}{\left\| x - x' \right\|^2} V(x,x')$$

Visibility term

$$V(x,x') = \begin{cases} 1 & \text{visible} \\ 0 & \text{not visible} \end{cases}$$

The Radiosity Equation

Assume diffuse reflection

1. $f_r(x,\omega_i \to \omega_o) = f_r(x) \Rightarrow \rho(x) = \pi f_r(x)$
2. $L(x,\omega) = B(x)/\pi$

$$B(x) = B_e(x) + \rho(x)E(x)$$

$$B(x) = B_e(x) + \rho(x) \int_{M^2} F(x,x')B(x')\, dA'(x')$$

$$F(x,x') = \frac{G(x,x')}{\pi}$$

Integral Equations

Integral equations of the 1st kind

$$f(x) = \int k(x,x')g(x')\,dx'$$

Integral equations of the 2nd kind

$$f(x) = g(x) + \int k(x,x')\,f(x')\,dx'$$

Linear Operators

Linear operators act on functions like matrices act on vectors

$$h(x) = (L \circ f)(x)$$

They are linear in that

$$L \circ (af + bg) = a(L \circ f) + b(L \circ g)$$

Types of linear operators

$$(K \circ f)(x) \equiv \int k(x,x')\,f(x')\,dx'$$

$$(D \circ f)(x) \equiv \frac{\partial f}{\partial x}(x)$$

Rendering Operators

Scattering

$$L_o(x,\omega_o) = S \circ L_i(x,\omega_i)$$

$$S \circ L_i(x,\omega_i) \equiv \int_{H^2} f_r(x,\omega_i \to \omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

Transport

$$L_i(x,\omega_i) = T \circ L_o(x,\omega_o)$$

$$T \circ L_o(x,\omega_o) \equiv L_o(x^*(x,\omega_o), -\omega_o)$$

Solving the Rendering Equation

Rendering Equation

$$K \equiv T \circ S$$

$$L = L_e + K \circ L$$

Solution

$$(I - K) \circ L = L_e$$

$$L = (I - K)^{-1} \circ L_e$$

Formal Solution

Neumann series

$$(I-K)^{-1} = \frac{1}{I-K} = I + K + K^2 + \ldots$$

Verify

$$\begin{aligned}(I-K)\circ(I-K)^{-1} &= (I-K)\circ\left(I+K+K^2+\ldots\right) \\ &= (I+K+\ldots)-\left(K+K^2+\ldots\right) \\ &= I\end{aligned}$$

Successive Approximations

Successive approximations

$$L^1 = L_e$$

$$L^2 = L_e + K\circ L^1$$

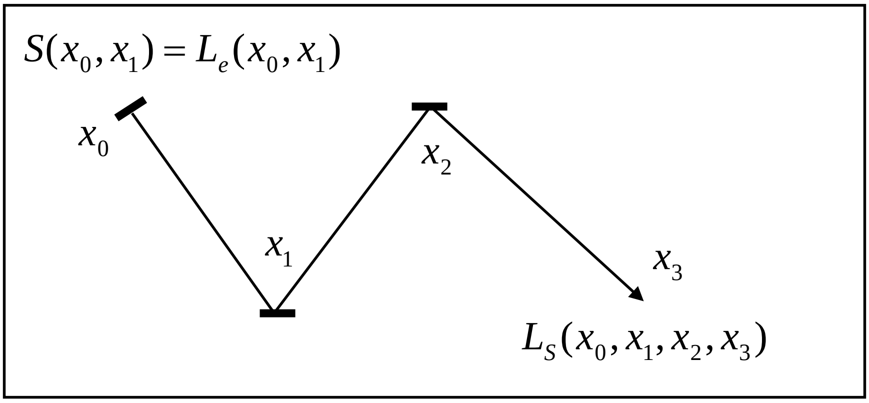
$$\ldots$$

$$L^n = L_e + K\circ L^{n-1}$$

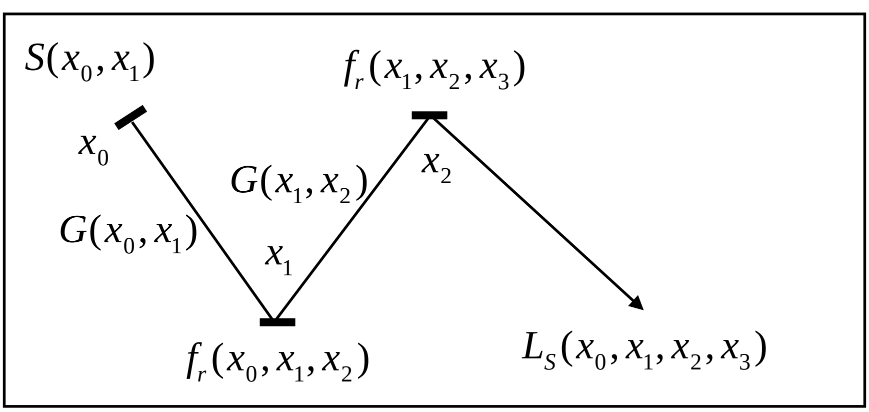
Converged

$$L^n = L^{n-1} \quad \therefore \quad L^n = L_e + K\circ L^n$$

Light Path

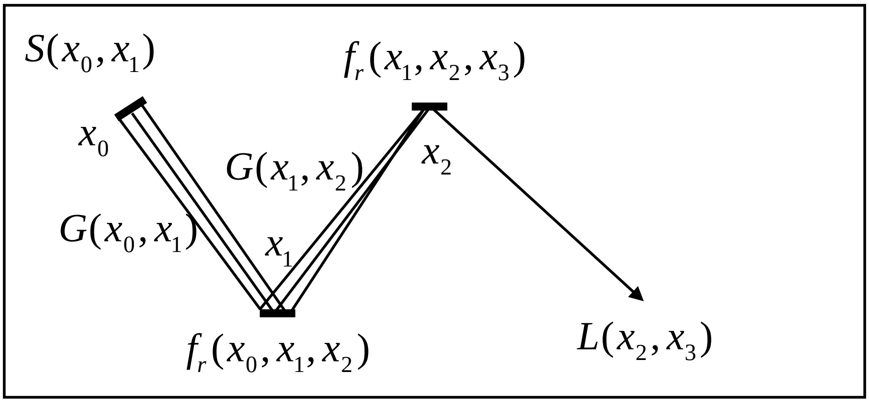


Light Path



$$L_S(x_0, x_1, x_2, x_3) = S(x_0, x_1) G(x_o, x_1) f_r(x_0, x_1, x_2) G(x_1, x_2) f_r(x_1, x_2, x_3)$$

Light Paths



$$L(x_2, x_3) = \int_{A_0} \int_{A_1} L_S(x_0, x_1, x_2, x_3) dA(x_0) dA(x_1)$$

Light Transport

Integrate over all paths of all lengths

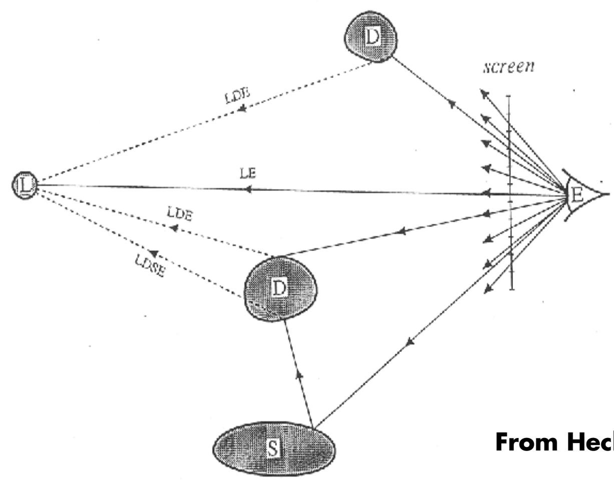
$$L(x_{k-1}, x_k)$$

$$= \sum_{k=1}^{\infty} \int_{M^2} \cdots \int_{M^2} L_S(x_0, \cdots, x_{k-2}, x_{k-1}, x_k) dA(x_0) \cdots dA(x_{k-2})$$

Questions

- **How to sample space of paths**
- **Find good estimators**

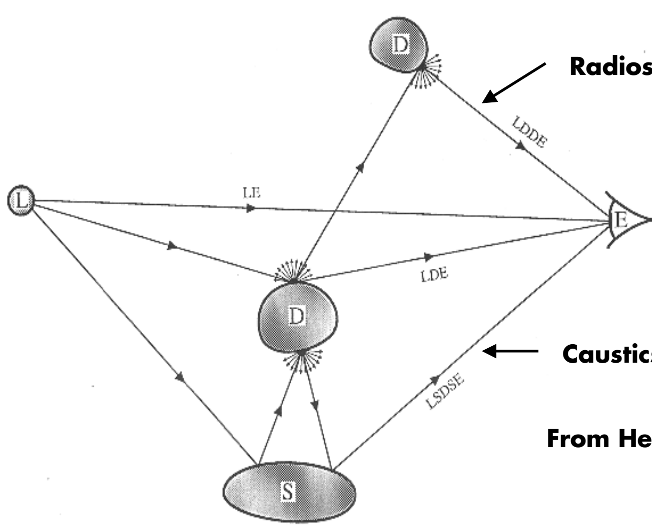
Classic Ray Tracing



From Heckbert

Forward (from eye): E S* (D|G) L

Photon Paths



Radiosity

Caustics

From Heckbert

How to Solve It?

Finite element methods

- Classic radiosity
  - Mesh surfaces
  - Piecewise constant basis functions
  - Solve matrix equation
- Not practical for rendering equation

Monte Carlo methods

- Distributed ray tracing
  - Randomly traces ray from the eye
- Path tracing
- Bidirectional ray tracing
- Photon tracing