Participating Media & Vol. Scattering

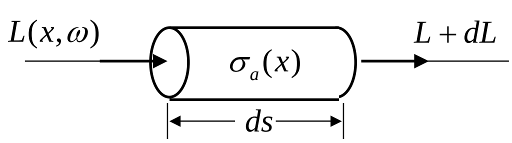
Applications

- Clouds, smoke, water, ...
- Subsurface scattering: paint, skin, ...
- Scientific/medical visualization: CT, MRI, ...

Topics

- Absorption and emission
- Scattering and phase functions
- Volume rendering equation
- Homogeneous media
- Ray tracing volumes

Absorption



$$dL(x,\omega) = -\sigma_a(x) L(x,\omega)\, ds$$

Absorption cross-section: $\sigma_a(x)$

Probability of being absorbed per unit length

CS348B Lecture 13 Pat Hanrahan, Spring 2002

Transmittance

$$dL(x,\omega) = -\sigma_a(x) L(x,\omega)\, ds$$

$$\frac{dL(x,\omega)}{L(x,\omega)} = -\sigma_a(x)\, ds$$

$$\ln L(x+s\,\omega,\omega) = -\int_0^s \sigma_a(x+s'\omega)ds' = -\tau(s)$$

Optical distance or depth

$$\tau(s) = \int_0^s \sigma_a(x+s'\omega)\,ds'$$

Homogenous media: constant σ_a

$$\sigma_a \to \tau(s) = \sigma_a s$$

Transmittance and Opacity

$$dL(x,\omega) = -\sigma_a(x) L(x,\omega)\, ds$$

$$\frac{dL(x,\omega)}{L(x,\omega)} = -\sigma_a(x)\, ds$$

$$\ln L(x+s\,\omega,\omega) = -\int_0^s \sigma_a(x+s'\omega)ds' = -\tau(s)$$

$$L(x+s\,\omega,\omega) = e^{-\tau(s)} L(x,\omega) = T(s) L(x,\omega)$$

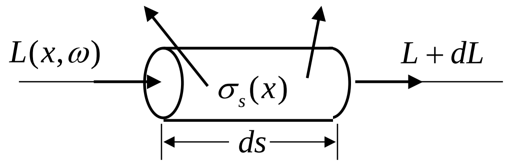
Transmittance

$$T(s) = e^{-\tau(s)}$$

Opacity

$$\alpha(s) = 1 - T(s)$$

Out-Scatter

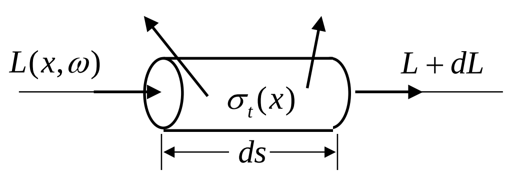


$$dL(x,\omega) = -\sigma_s(x) L(x,\omega)\, ds$$

Scattering cross-section: σ_s

Probability of being absorbed per unit length

Extinction



$$dL(x,\omega) = -\sigma_t(x) L(x,\omega)\, ds$$

Total cross-section $\sigma_t = \sigma_a + \sigma_s$

Albedo $W = \dfrac{\sigma_s}{\sigma_t} = \dfrac{\sigma_s}{\sigma_a + \sigma_s}$

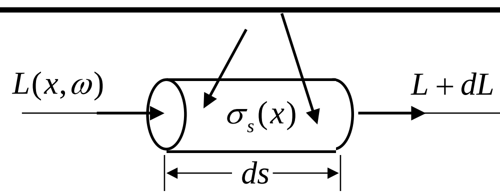
Attenuation due to both absorption and scattering

$$\tau(s) = \int_0^s \sigma_t(x + s'\omega)\, ds'$$

Black Clouds

From Greenler, Rainbows, halos and glories

In-Scatter



$$S(x,\omega) = \sigma_s(x) \int_{S^2} p(\omega' \to \omega) L(x,\omega')\, d\omega'$$

Phase function $p(\omega' \to \omega)$

Reciprocal

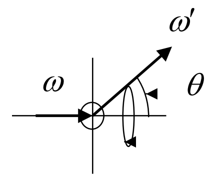
$$p(\omega \to \omega') = p(\omega' \to \omega)$$

Normalized

$$\int_{S^2} p(\omega' \to \omega)\, d\omega' = 1$$

Phase Functions

Phase angle $\cos\theta = \omega \bullet \omega'$



Phase functions
(from the phase of the moon)

1. **Isotropic**
   -simple
2. **Rayleigh**
   -molecules
3. **Mie scattering**
   - small spheres

... Huge literature ...

$$p(\cos\theta) = \frac{1}{4\pi}$$

$$p(\cos\theta) = \frac{3}{4}\frac{1+\cos^2\theta}{\lambda^4}$$

Blue Sky = Red Sunset

From Greenler, Rainbows, halos and glories

Coronas and Halos

Moon Corona

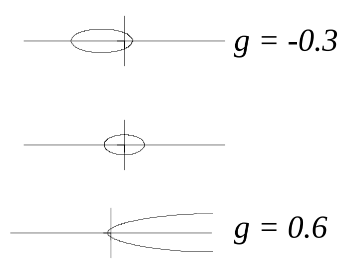
Sun Halos

From Greenler, Rainbows, halos and glories

Henyey-Greenstein Phase Function

Empirical phase function

$$p(\cos\theta) = \frac{1}{4\pi}\frac{1-g^2}{\left(1+g^2-2g\cos\theta\right)^{3/2}}$$



$$2\pi\int_0^\pi p(\cos\theta)\cos\theta\, d\theta = g$$

g: average phase angle

The Volume Rendering Equation

Integro-differential equation

$$\frac{\partial L(x,\omega)}{\partial s} = -\sigma_t(x)L(x,\omega) + S(x,\omega)$$

Integro-integral equation

$$L(x,\omega) = \int_0^\infty e^{-\int_0^{s'} \sigma_t(x+s''\omega)\,ds''} S(x+s'\omega)\,ds'$$

Attenuation: Absorption and scattering

Source: Scatter (+ emission)

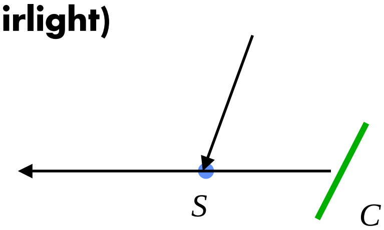
Simple Atmosphere Model

Assumptions

- **Homogenous media**
- **Constant source term (airlight)**

$$\frac{\partial L(s)}{\partial s} = -\sigma_t L(s) + S$$

$$L(s) = \left(1 - e^{-\sigma_t s}\right) S \quad + e^{-\sigma_t s} C$$



Fog

Haze

The Sky

Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)

Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

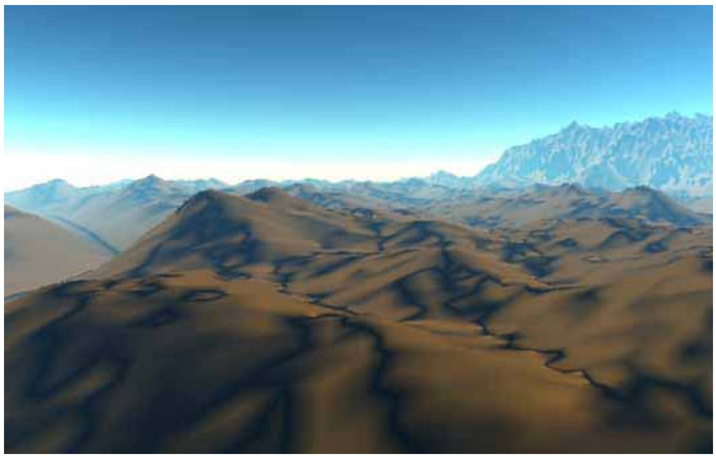
From Greenler, Rainbows, halos and glories

Atmospheric Perspective

From Greenler, Rainbows, halos and glories

Atmospheric Perspective

Aerial Perspective: loss of contrast and change in color



From Musgrave

Semi-Infinite Homogenous Media

Reduced Intensity

$$L(z,\omega_i) = e^{-\tau(z,\omega_i)} L(0,\omega_i)$$

Effective source term

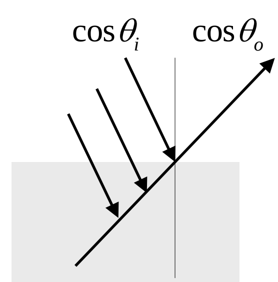
$$S(z,\omega_o) = \sigma_s p(\omega_i \to \omega_o) e^{-\tau(z,\omega_i)} L(0,\omega_i)$$

Volume rendering equation

$$\cos\theta_o \frac{\partial L(z,\omega_o)}{\partial z} = -\sigma_t L(z,\omega_o) + S(z,\omega_o)$$

Integrating over depths

$$\cos\theta_o L(\omega_o) = \int_0^\infty e^{-\sigma_t z/\cos\theta_o} \sigma_s p(\omega_i,\omega_o) e^{-\sigma_t z/\cos\theta_i} L(\omega_i)\, dz$$

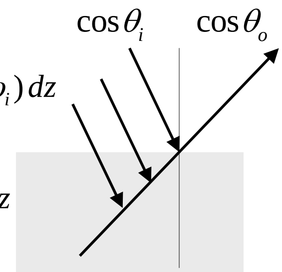


$$z = s\cos\theta$$

$$dz = ds\cos\theta$$

Semi-Infinite Homogenous Media

Integrating over depths



$$\cos\theta_o L(\omega_o) = \int_0^\infty e^{-\sigma_t z/\cos\theta_o} \sigma_s p(\omega_i,\omega_o) e^{-\sigma_t z/\cos\theta_i} L(\omega_i)\, dz$$

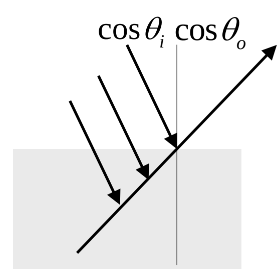
$$= \sigma_s p(\omega_i,\omega_o) L(\omega_i) \int_0^\infty e^{-\sigma_t\left[\frac{1}{\cos\theta_i}+\frac{1}{\cos\theta_o}\right]z}\, dz$$

$$= \sigma_s p(\omega_i,\omega_o) L(\omega_i) \frac{1}{\sigma_t\left[\frac{1}{\cos\theta_i}+\frac{1}{\cos\theta_o}\right]}$$

$$= W\, p(\omega_i,\omega_o)\, L(\omega_i) \frac{\cos\theta_i \cos\theta_o}{\cos\theta_i + \cos\theta_o}$$

Semi-Infinite Homogenous Media

BRDF



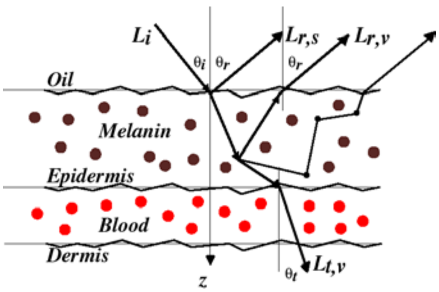
$$f_r(\omega_i,\omega_o) = \frac{dL}{dE} = \frac{L(\omega_i,\omega_o)}{L(\omega_i)\cos\theta_i}$$

$$= W\, p(\omega_i,\omega_o) \frac{1}{\cos\theta_i + \cos\theta_o}$$

Seeliger's Law or The Law of Diffuse Reflection

Subsurface Scattering

Skin



Volume Representations

3D arrays (uniform rectangular)

- CT data

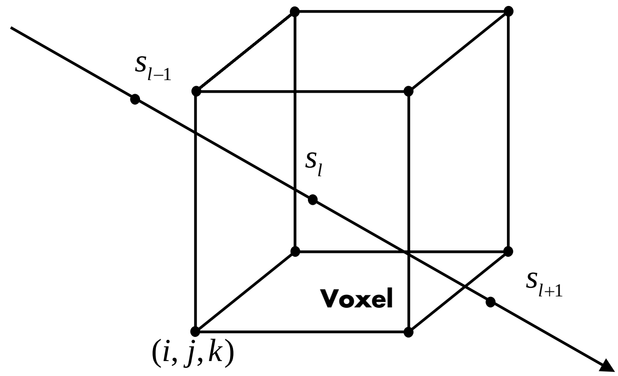
3D meshes

- CFD, mechanical simulation

Simple shapes with solid texture

- Ellipsoidal clouds with sum-of-sines densities
- Hypertexture

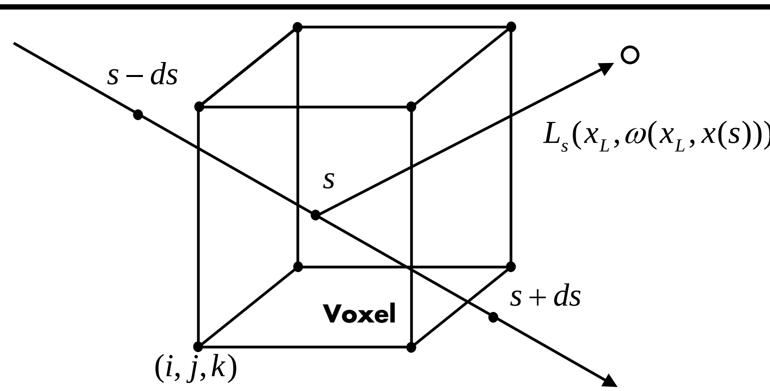
Scalar Volumes



Interpolation $v(s_l) = \text{trilinear}(v, i, j, k, x(s_l))$

Map scalars to optical properties $\sigma_s(v), \sigma_a(v)$

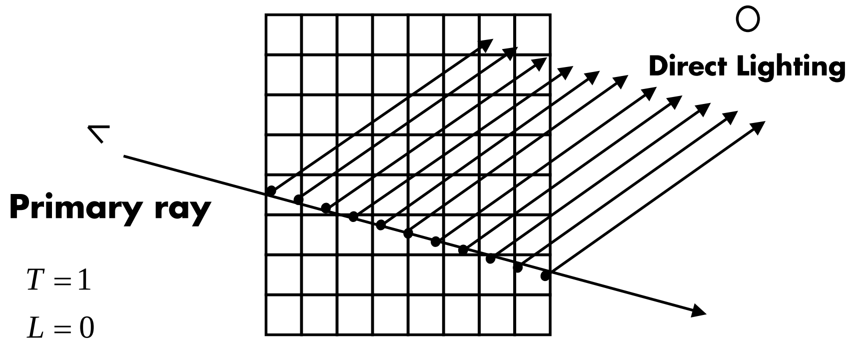
Scalar Volumes



Scatter

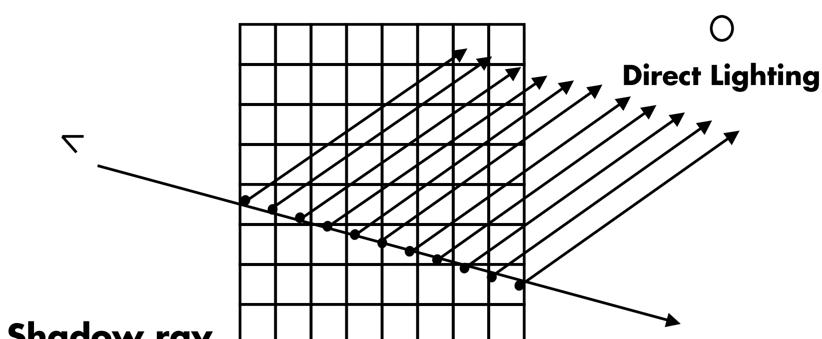
$$S(x(s),\omega) = \sigma_s(s)\, p(\omega, \omega(x(s), x_L))\, L_s(x_L, \omega(x_L, x(s)))$$

Ray Marching



$$T = 1$$
$$L = 0$$
$$\text{for}(s = 0;\, s < 1;\, s+ = ds)$$
$$\quad S = \sigma_s(s)\, p(\omega, \omega(x(s), x_L))\, L_s(x_L, \omega(x_L, x(s)))$$
$$\quad L = L + TS\Delta s$$
$$\quad T = T\left[1 - \sigma_t(x(s))\right]\Delta s$$

Ray Marching



$$T = 1$$
$$\text{for}(t = 0;\, t < 1;\, t+ = dt)$$
$$\quad T = T\left[1 - \sigma_t(x(t))\right]\Delta t$$
$$S(x(s)) = \sigma_s(s)\, p(\omega, \omega(x(s), x_L))\, T L_s(x_L, \omega(x_L, x(s)))$$

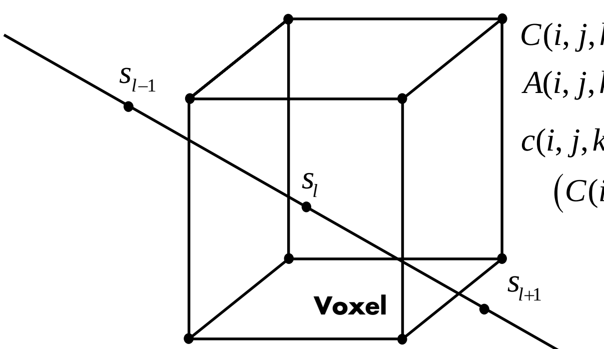
Beams of Light

From Greenler, Rainbows, halos and glories

From Minneart, Color and light in the open air

Color and Opacity Volumes

M. Levoy, Ray tracing volume densities



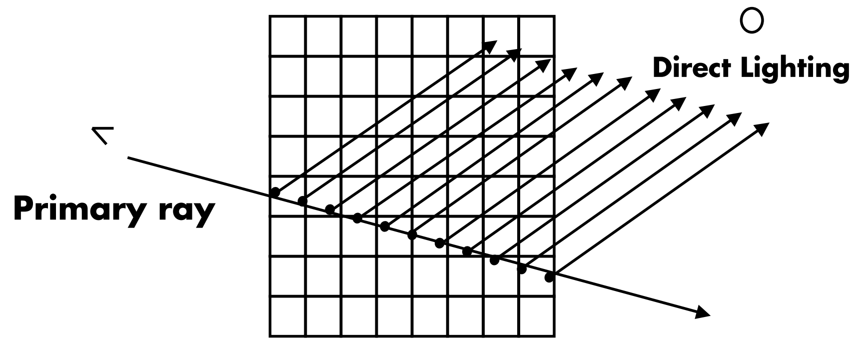
$$C(i,j,k) \Rightarrow (R,G,B)$$

$$A(i,j,k)$$

$$c(i,j,k) = \big(C(i,j,k) * A(i,j,k), A(i,j,k)\big)$$

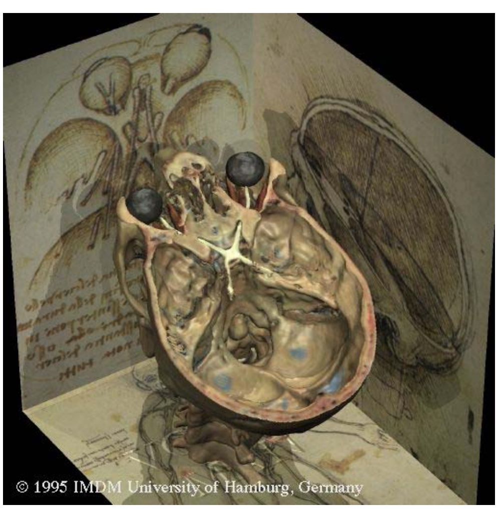
$$c(x(s_l)) = \text{trilinear}(c,i,j,k,x(s_l))$$

Ray Marching



$$C = (0,0,0,0)$$
$$\text{for}(s = 0; s < 1; s += ds)$$
$$C = C + (1 - \alpha(C))c(s)$$

Volume Rendering Examples



From Karl Heinz Hoehne

From Marc Levoy