Reflection Models

Last lecture

- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

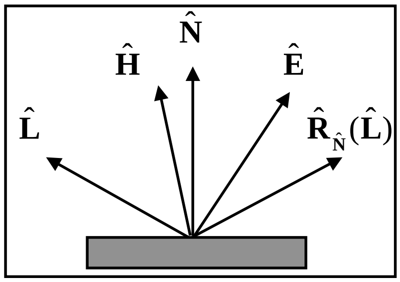
Today

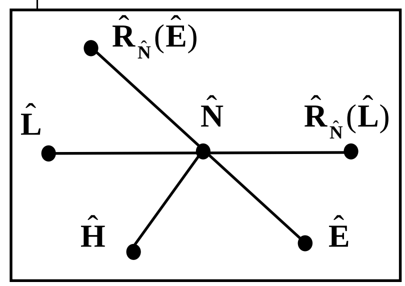
- Phong and mircofacet models
- Gaussian height surface
- Self-shadowing
- Torrance-Sparrow model
- Anisotropic reflection models

Phong Model

Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$$





$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

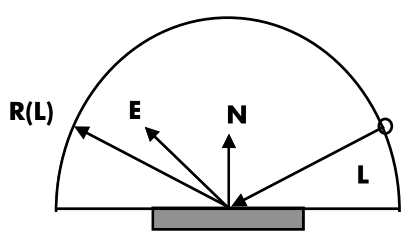
$$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$$

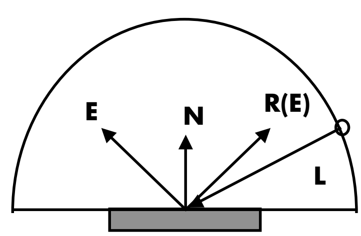
$$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$$

$$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$$

$$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$$

Phong Model



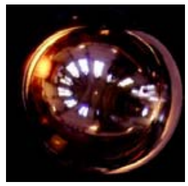


$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s \qquad \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$$

Reciprocity: $\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

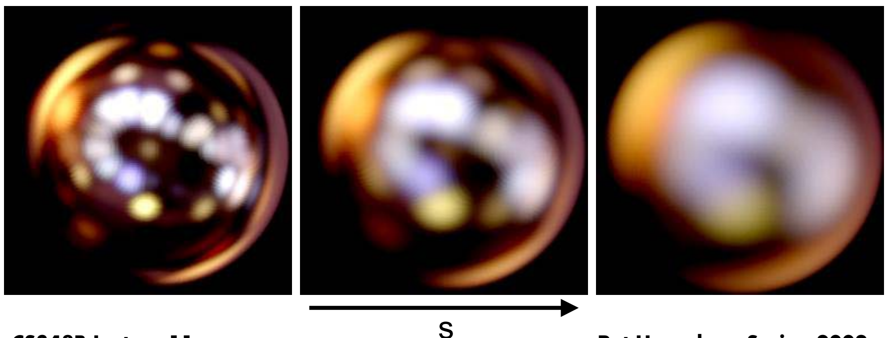
Distributed light source!

Phong Model



Mirror

Diffuse



Energy normalization

Energy normalize Phong Model

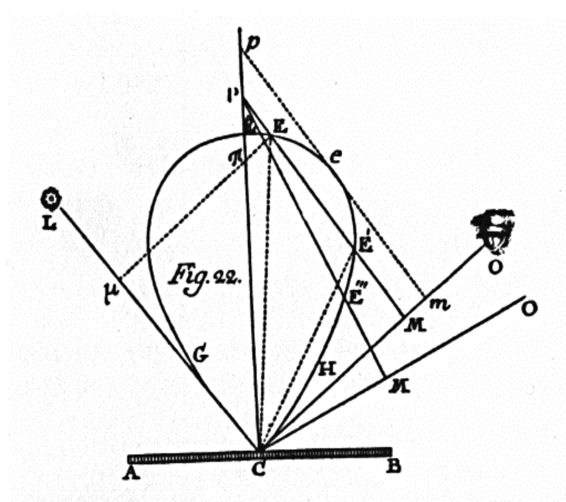
$$\rho(H^2 \to \omega_r) = \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s \cos\theta_i \, d\omega_i$$

$$\leq \int_{H^2(\hat{\mathbf{R}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_{ir}$$

$$\leq \int_{H^2} \cos^s \theta \, d\omega = \frac{2\pi}{s+1}$$

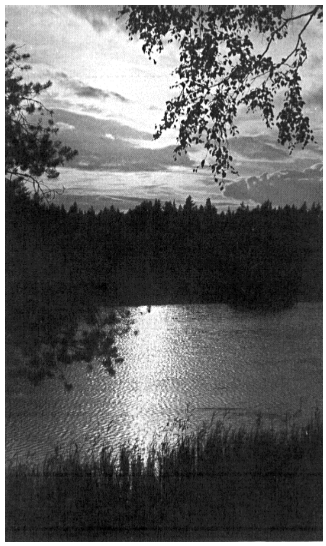
Microfacet Model

Bouguer's "little faces"

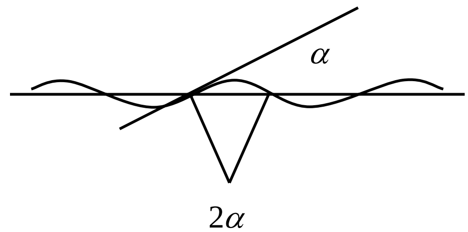


P. Bouguer, *Treatise on Optics*, 1760

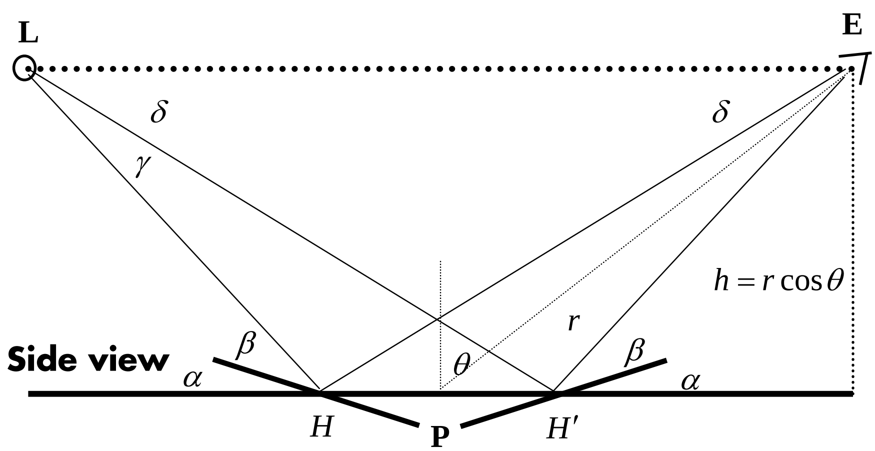
Reflection of the Sun from the Sea



Source: Minnaert, *Light and Color in Outdoors*, p. 28



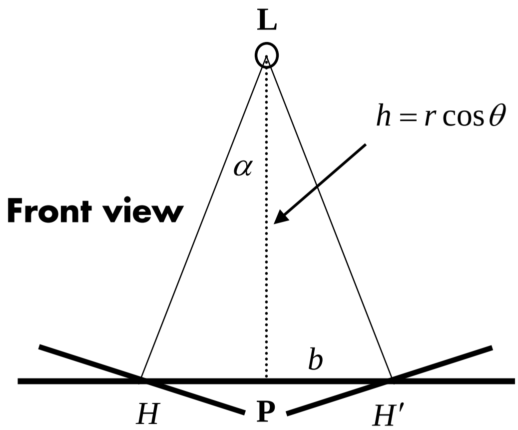
Reflection Angles



Assume L and E are at the same height h

$$\alpha + \beta = \gamma + \delta$$
$$\beta - \alpha = \delta$$
$$\Rightarrow \quad \gamma = 2\alpha$$

Reflection Angles



$$\tan\alpha = \frac{b}{h}$$

$$\tan\psi = \frac{b}{r} = \frac{h}{r}\tan\alpha = \tan\alpha\cos\theta$$

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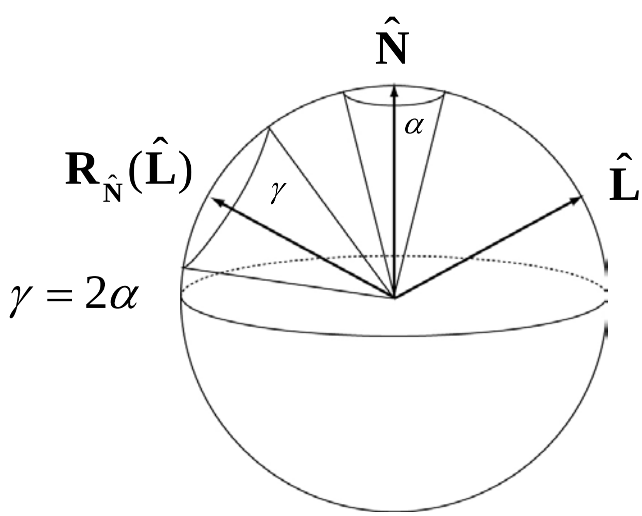
Analysis on the Sphere



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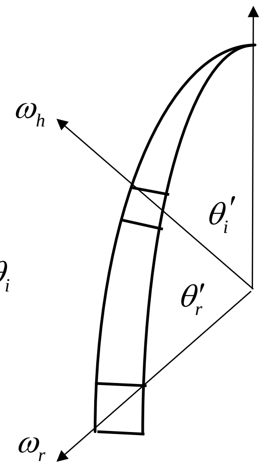
Solid Angle Distributions



$\theta_i = \theta_i'$

$\theta_r' = \theta_i'$

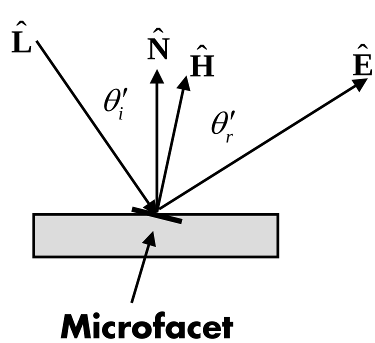
$\theta_r = 2\theta_i$

$\varphi_r = \varphi_i$

$$
\begin{aligned}
d\omega_r &= \sin\theta_r \, d\theta_r \, d\varphi_r \\
&= \left(\sin 2\theta_i\right) 2 d\theta_i \, d\varphi_i \\
&= \left(2\sin\theta_i \cos\theta_i\right) 2 d\theta_i \, d\varphi_i \\
&= 4\cos\theta_i \, d\omega_h
\end{aligned}
$$

$$\frac{d\omega_h}{d\omega_r} = \frac{1}{4\cos\theta_i'}$$

Microfacet Distributions



Normalization

$$\int_{H^2} \cos\theta_h \, dA(\omega_h) = dA$$

$$\int_{H^2} D(\omega_h) \cos\theta_h \, d\omega_h = 1$$

$$dA(\omega_h)\, d\omega_h = D(\omega_h)\, d\omega_h \, dA$$

Microfacet Distribution Functions

Isotropic distributions $D(\omega_h) \Rightarrow D(\alpha)$

Characterize by half-angle β $D(\beta) = \frac{1}{2}$

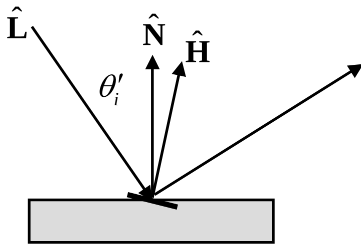
Examples:

- **Blinn** $D_1(\alpha) = \cos^{c_1} \alpha \qquad c_1 = \frac{\ln 2}{\ln \cos \beta}$
- **Torrance-Sparrow** $D_2(\alpha) = e^{-(c_2 \alpha)^2} \qquad c_2 = \frac{\sqrt{2}}{\beta}$
- **Trowbridge-Reitz** $D_3(\alpha) = \frac{c_3^2}{(1-c_3^2)\cos^2 \alpha - 1}$

$$c_3 = \left(\frac{\cos^2 \beta - 1}{\cos^2 \beta - \sqrt{2}} \right)^{1/2}$$

Torrance-Sparrow Model

Torrance-Sparrow Model



$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$
$$\cos\theta_i' = \hat{\mathbf{L}} \bullet \hat{\mathbf{H}}$$

$$\begin{aligned} d\Phi_h &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, dA(\omega_h) \\ &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA \end{aligned}$$

$$dA(\omega_h)\, d\omega_h = D(\omega_h)\, d\omega_h\, dA$$

$$d\Phi_r = dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$

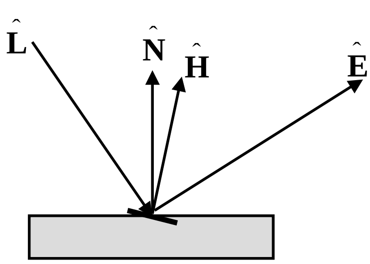
$$d\Phi_r = d\Phi_h$$

$$\begin{aligned} \therefore\ & dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA \\ &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA \end{aligned}$$

Torrance-Sparrow Model

$$\begin{aligned} & dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA \\ &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA \end{aligned}$$

$$\begin{aligned} f_r(\omega_i \to \omega_r) &\equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i(\omega_i)} \\ &= \frac{L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA}{\left(\cos\theta_r\, d\omega_r\, dA\right)\left(L_i(\omega_i)\cos\theta_i\, d\omega_i\right)} \\ &= \frac{D(\omega_h)}{\cos\theta_i\cos\theta_r}\cos\theta_i' \frac{d\omega_h}{d\omega_r} \\ &= \frac{D(\omega_h)}{4\cos\theta_i\cos\theta_r} \end{aligned}$$



$$d\omega_i' = d\omega_i$$

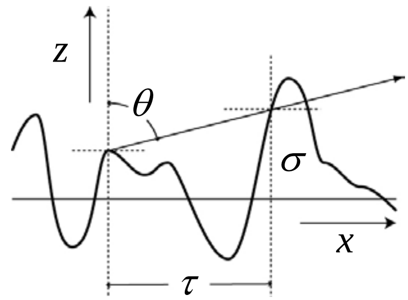
Self-Shadowing

Shadows on Rough Surfaces

Gaussian Rough Surface

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$

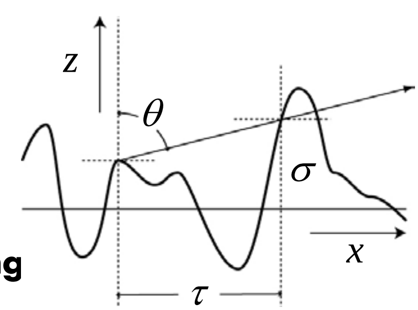


Gaussian distribution of slopes

$$D(\alpha) = \frac{1}{\sqrt{\pi} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}} \qquad m = \frac{2\sigma}{\tau}$$

Beckmann

Self-Shadowing Function

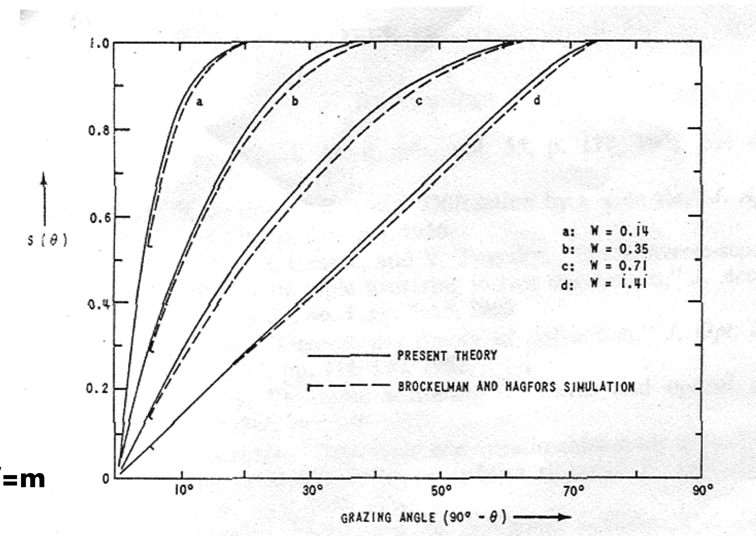


Probability of shadowing

$$S(\theta) = \frac{\left[1 - \frac{1}{2}\operatorname{erfc}\left(\mu/\sqrt{2}m\right)\right]}{1 + \Lambda(\mu)}$$

$$2\Lambda(\mu) = \left(\sqrt{\frac{2}{\pi}}\right)\frac{m}{\mu} e^{-\mu^2/2m^2} - \operatorname{erfc}\left(\mu/\sqrt{2}m\right)$$

Self-Shadowing Function



W=m

From Smith, 1967

Self-Consistency Condition

$$\int S(\theta) D(\alpha) \cos\theta' \, d\omega_\alpha = \cos\theta$$

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

Torrance-Sparrow Experiments

$$f_r(\omega_i \to \omega_r)$$
$$= \frac{F(\theta_i')S(\theta_i)S(\theta_r)D(\alpha)}{4\cos\theta_i \cos\theta_r}$$

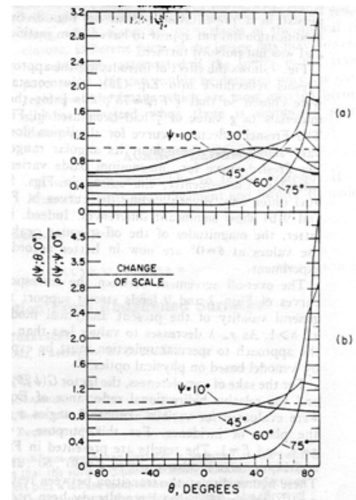
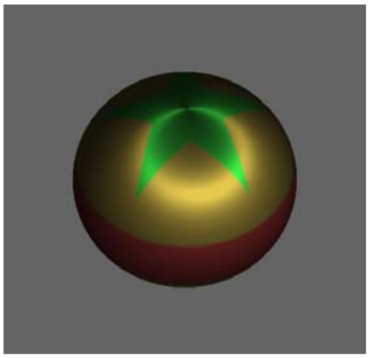


FIG. 9. Predicted bidirectional reflectance distributions corresponding to the experimental distributions in Fig. 2. Calculated from Eq. (23) with $c=0.05$ and $F(\psi', \hat{n})$ evaluated at $\lambda = 0.5\ \mu$. (a) Aluminum, $g=\frac{3}{2}$. (b) Magnesium oxide, $g=2$.

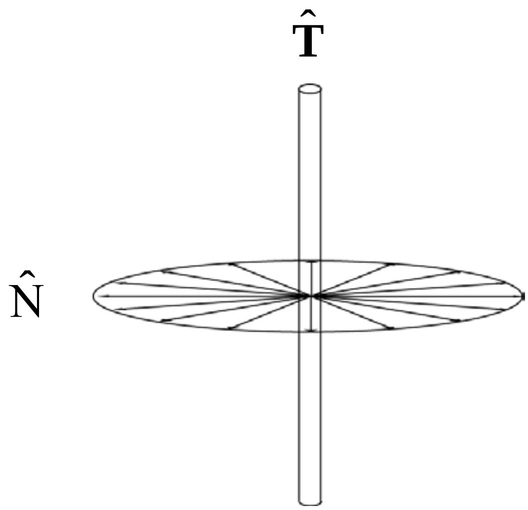
Anisotropic Reflection Model

Anisotropic Reflection

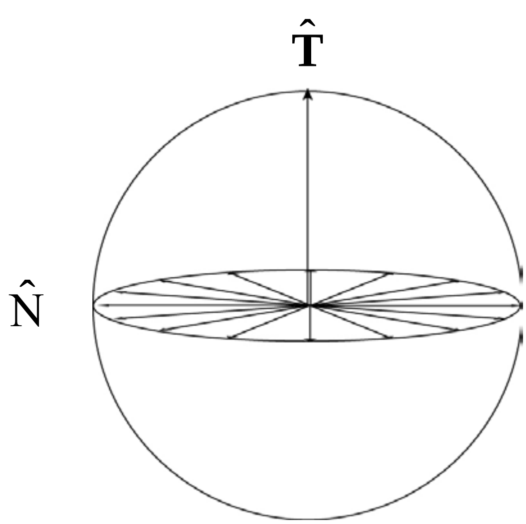


Quarterhorse

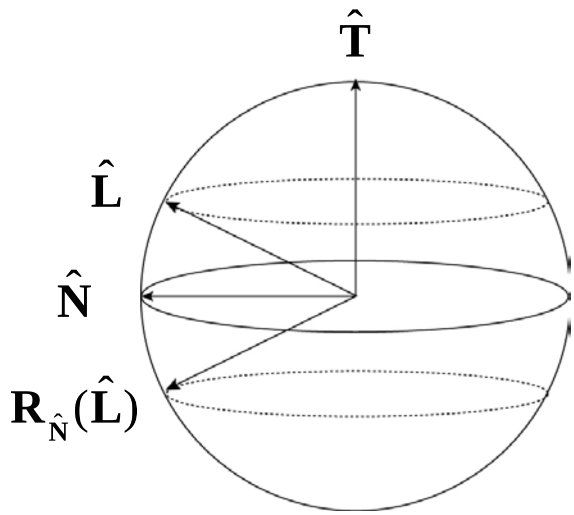
Reflection from a Cylinder



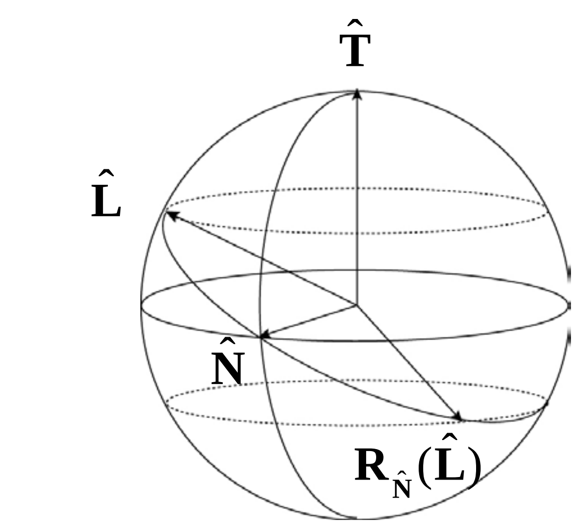
Reflection from a Cylinder



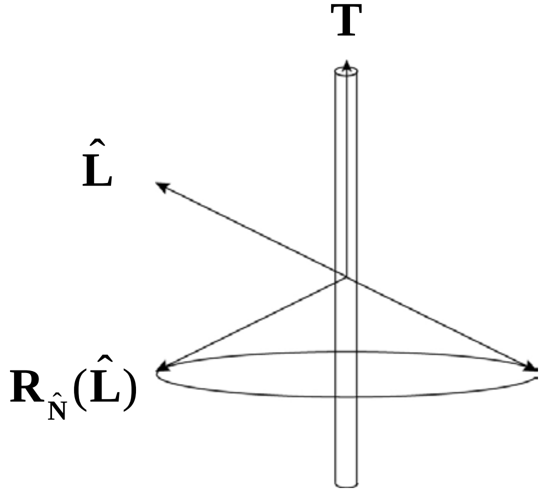
Reflection from a Cylinder



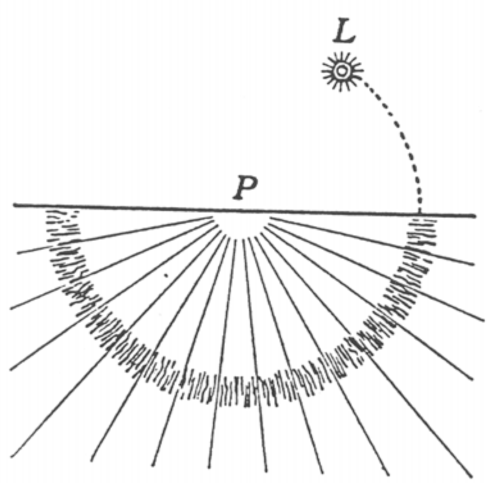
Reflection from a Cylinder



Reflection from a Cylinder

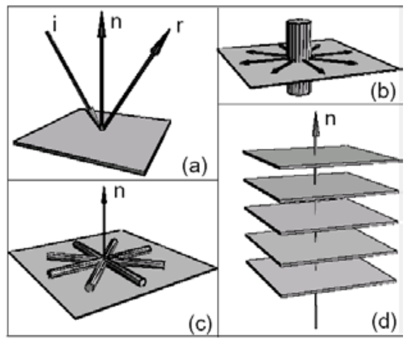


Anisotropic Reflection



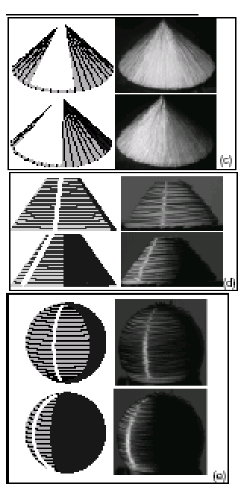
Shape of Anisotropic Highlights

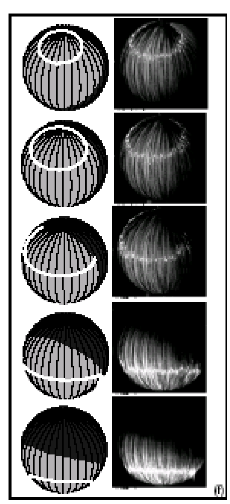
Fibers tangent to the plane defined by the halfway vector reflect light



From Lu, Koenderink, Kappers

Shape of Anisotropic Highlights



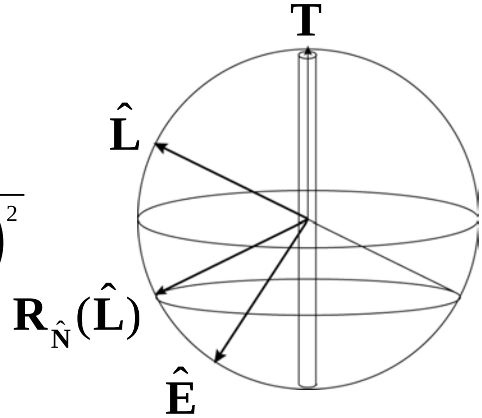


From Lu, Koenderink, Kappers

Kay-Kajiya Model

Diffuse

$$\sin\theta_L = \sqrt{1-\left(\hat{\mathbf{T}}\bullet\hat{\mathbf{L}}\right)^2}$$



Specular

$$\cos^s\left(\theta_E - \theta_L\right) = \left(\cos\theta_E \cos\theta_L + \sin\theta_E \sin\theta_L\right)^s$$

Herbert

Hair Model

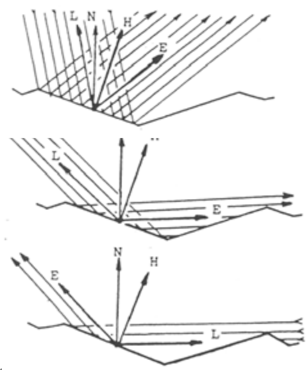
Black Hair **Brown Hair**

Self-Shadowing V-Groove Model

Self-Shadowing: V-Groove Model

Assumptions (Torrance-Sparrow)

1. **Symmetric, longitudinal, isotropically-distributed**
2. **Upper edges lie in plane**



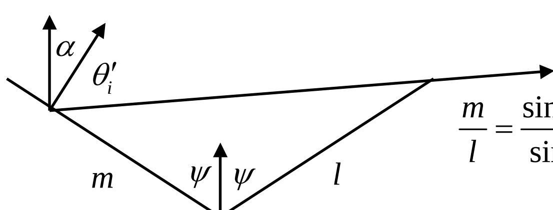
$$G = \min(G_a, G_b, G_c)$$

$$G_a = 1$$

$$G_b = \frac{2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}})}{(\hat{\mathbf{H}} \bullet \hat{\mathbf{E}})}$$

$$G_c = \frac{2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{L}})}{(\hat{\mathbf{H}} \bullet \hat{\mathbf{L}})}$$

Self-Shadowing: V-Groove Model



$$\frac{m}{l} = \frac{\sin m}{\sin l}$$

$$\sin l = \cos\theta'_i$$
$$\cos l = \sin\theta'_i$$
$$\sin\psi = \cos\alpha$$
$$\cos\psi = \sin\alpha$$

$$\begin{aligned}
\sin m &= \sin l + 2\psi \\
&= \sin l \cos 2\psi + \cos l \sin 2\psi \\
&= \cos\theta'_i \cos 2\psi + \sin\theta'_i \sin 2\psi \\
&= \cos\theta'_i (1 - 2\sin^2\psi) + \sin\theta'_i 2\cos\psi\sin\psi \\
&= \cos\theta'_i (1 - 2\cos^2\alpha) + \sin\theta'_i 2\cos\alpha\sin\alpha \\
&= \cos\theta'_i - 2\cos\alpha\left(\cos\alpha\cos\theta'_i - \sin\alpha\sin\theta''_i\right) \\
&= \cos\theta'_i - 2\cos\alpha\cos\left(\alpha + \theta'_i\right) \\
&= \cos\theta'_i - 2\cos\alpha\cos\theta_r \\
&= \hat{\mathbf{H}} \bullet \hat{\mathbf{E}} - 2\left(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}}\right)\left(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}}\right)
\end{aligned}$$

$$\begin{aligned}
G &= 1 - \frac{m}{l} \\
&= 1 - \frac{\sin m}{\sin l} \\
&= \frac{\hat{\mathbf{H}} \bullet \hat{\mathbf{E}} - \hat{\mathbf{H}} \bullet \hat{\mathbf{E}} + 2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}})}{\hat{\mathbf{H}} \bullet \hat{\mathbf{E}}} \\
&= \frac{2(\hat{\mathbf{N}} \bullet \hat{\mathbf{H}})(\hat{\mathbf{N}} \bullet \hat{\mathbf{E}})}{\hat{\mathbf{H}} \bullet \hat{\mathbf{E}}}
\end{aligned}$$