Reflection Models

Tuesday

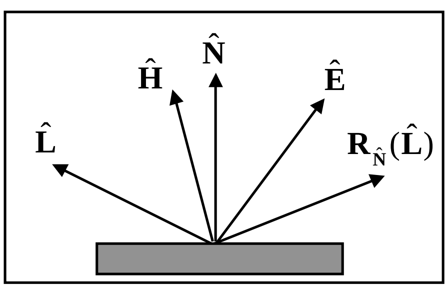
- **Reflection models**
- **The reflection equation and the BRDF**
- **Ideal reflection, refraction and diffuse**

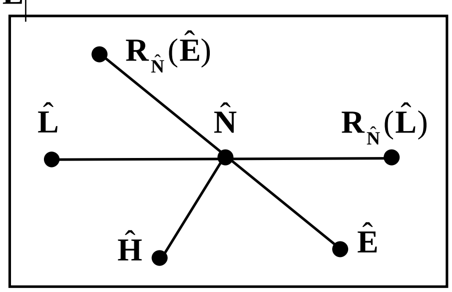
Today

- **Glossy reflection models**
- **Rough surfaces**
- **Microfacets**
- **Self-shadowing**
- **Fresnel effects**
- **Anisotropic reflection models**

Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$$





$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

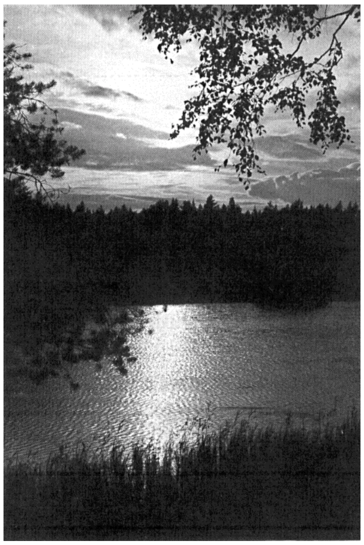
$$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$$

$$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$$

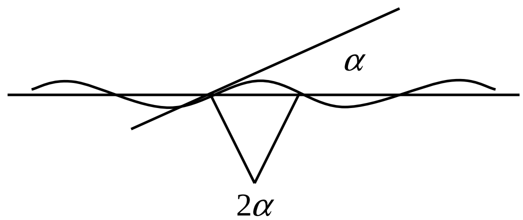
$$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$$

$$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$$

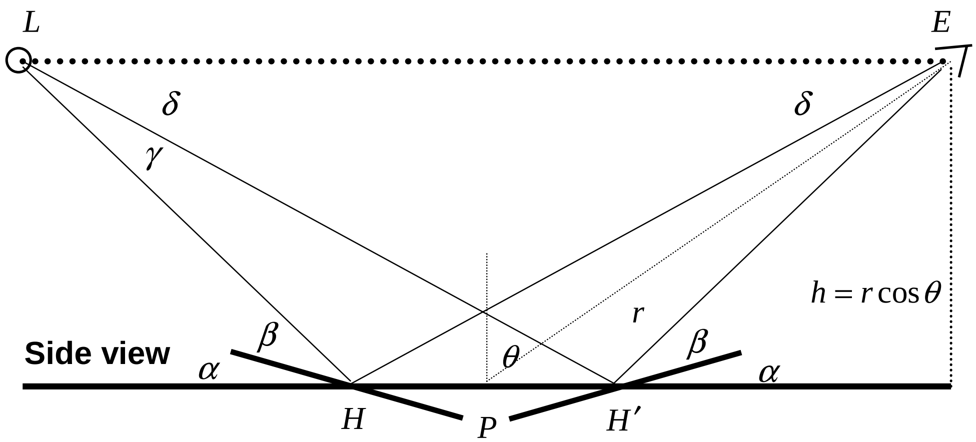
Reflection of the Sun from the Sea



Source: Minnaert, *Light and Color in the Outdoors*, p. 28



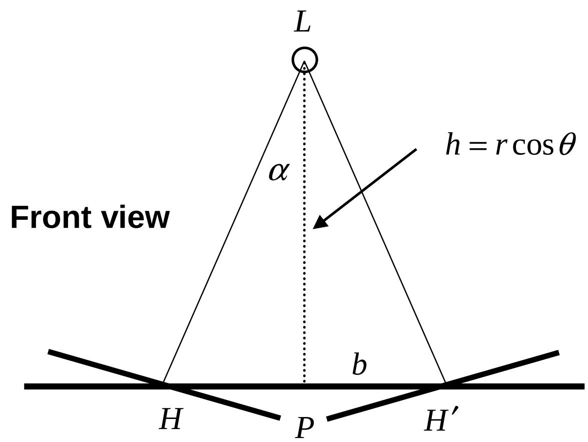
Reflection Angles



Assume *L* and *E* are at the same height

$$\alpha + \beta = \gamma + \delta$$
$$\beta - \alpha = \delta$$
$$\Rightarrow \quad \gamma = 2\alpha$$

Reflection Angles

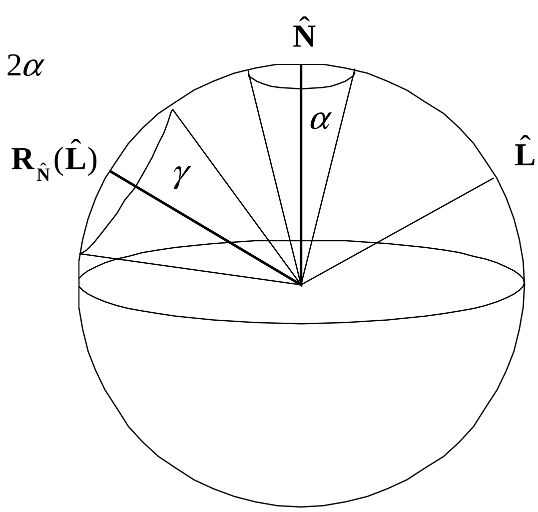


$$\tan\psi = \frac{b}{r} = \frac{b}{h/\cos\theta} = \tan\alpha\cos\theta$$

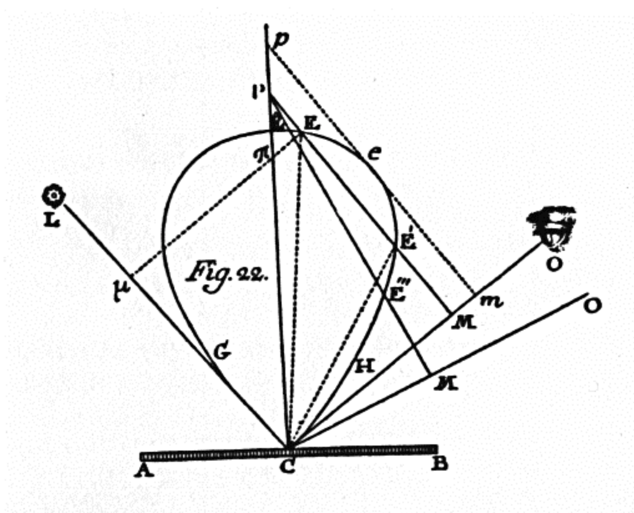
$$\tan\alpha = \frac{b}{h}$$

Analysis on the Sphere

$$\gamma = 2\alpha$$

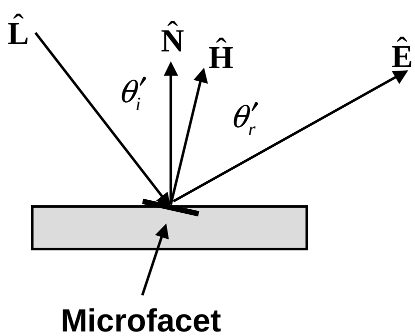


Bouguer's "little faces"



P. Bouguer, ***Treatise on Optics***, 1760

Microfacet Model



$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

$$\cos\theta_i' = \hat{\mathbf{L}} \bullet \hat{\mathbf{H}}$$

$$d\omega_i' = d\omega_i$$

$$dA' = D(\omega_h)\, d\omega_h\, dA$$

$$dA(\omega_h)\, d\omega_h = D(\omega_h)\, d\omega_h\, dA$$

$$\int_{H^2} \cos\theta_h\, dA' = \int_{H^2} D(\omega_h)\, d\omega_h\, dA = dA$$

$$\int_{H^2} D(\omega_h) \cos\theta_h\, d\omega_h = 1$$

Microfacet Distribution Functions

Isotropic distributions $D(\omega) \Rightarrow D(\alpha)$

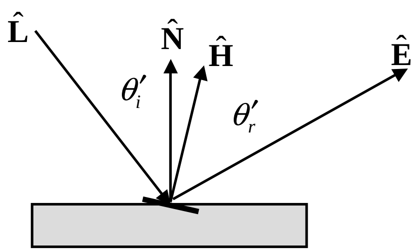
Characterize by half-angle β $D(\beta) = \frac{1}{2}$

Examples:

- **Blinn** $D_1(\alpha) = \cos^{c_1} \alpha$ $c_1 = \frac{\ln 2}{\ln \cos \beta}$
- **Torrance-Sparrow** $D_2(\alpha) = e^{-(c_2 \alpha)^2}$ $c_2 = \frac{\sqrt{2}}{\beta}$
- **Trowbridge-Reitz** $D_3(\alpha) = \frac{c_3^2}{(1-c_3^2)\cos^2 \alpha - 1}$

$$c_3 = \left(\frac{\cos^2 \beta - 1}{\cos^2 \beta - \sqrt{2}} \right)^{1/2}$$

Torrance-Sparrow Model



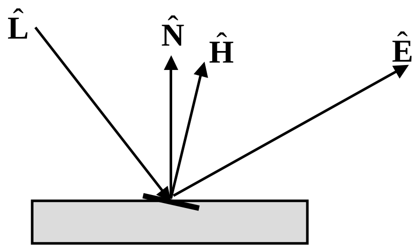
$$dA(\omega_h)\, d\omega_h = D(\omega_h)\, d\omega_h\, dA$$

$$d\Phi_r = d\Phi_h$$

$$d\Phi_h = L_i(\omega_i) \cos \theta_i' \, d\omega_i' \, dA'$$
$$= L_i(\omega_i) \cos \theta_i' \, d\omega_i \, D(\omega_h)\, d\omega_h\, dA$$
$$d\Phi_r = dL_r(\omega_i \to \omega_r) \cos \theta_r \, d\omega_r \, dA$$

$$\therefore dL_r(\omega_i \to \omega_r) \cos \theta_r \, d\omega_r \, dA$$
$$= L_i(\omega_i) \cos \theta_i' \, d\omega_i \, D(\omega_h)\, d\omega_h\, dA$$

Torrance-Sparrow Model



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i(\omega_i)}$$

$$= \frac{L_i(\omega_i)\cos\theta_i'\, d\omega_i\; D(\omega_h)\, d\omega_h\, dA}{(\cos\theta_r d\omega_r dA)(L_i(\omega_i)\cos\theta_i\, d\omega_i)}$$

$$= \frac{D(\omega_h)}{\cos\theta_i \cos\theta_r}\cos\theta_i' \frac{d\omega_h}{d\omega_r}$$

$$= \frac{D(\omega_h)}{4\cos\theta_i \cos\theta_r}$$

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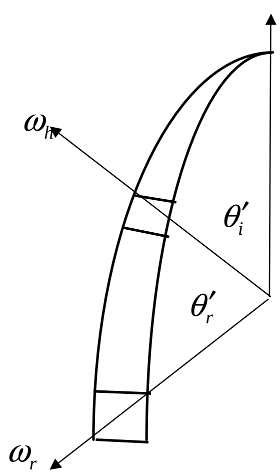
Solid Angle Distributions

$$\theta_i = \theta_i'$$

$$\theta_r' = \theta_i'$$

$$\theta_r = 2\theta_i$$

$$\varphi_r = \varphi_i$$



$$d\omega_r = \sin\theta_r\, d\theta_r\, d\varphi_r$$

$$= (\sin 2\theta_i)\, 2d\theta_i\, d\varphi_i$$

$$= (2\sin\theta_i \cos\theta_i)\, 2d\theta_i\, d\varphi_i$$

$$= 4\cos\theta_i\, d\omega_h$$

$$\frac{d\omega_h}{d\omega_r} = \frac{1}{4\cos\theta_i'}$$

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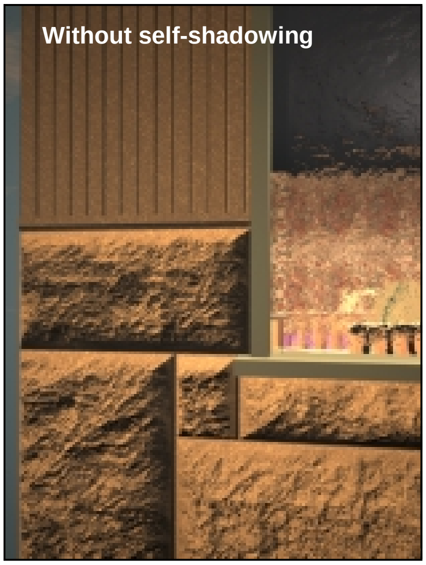
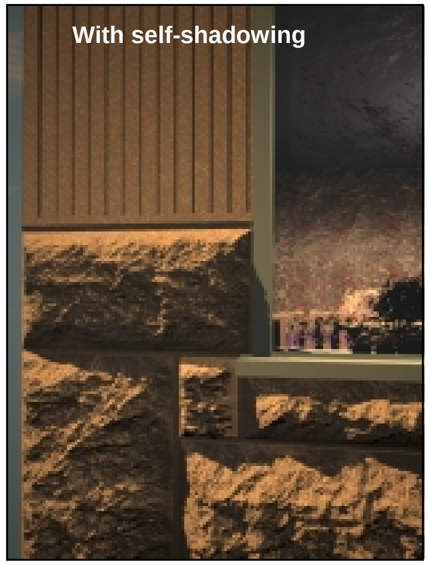
Shadows on Rough Surfaces

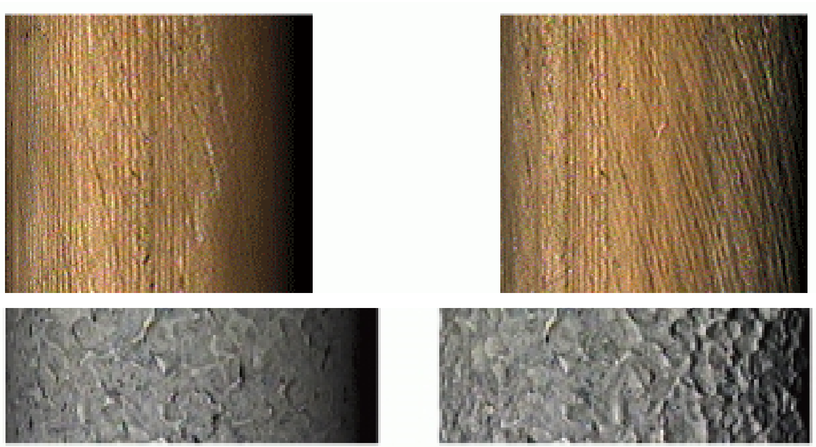




S. Nayar's BTF Experiments

Complex interplay between texture and brdf [Better examples from db!]

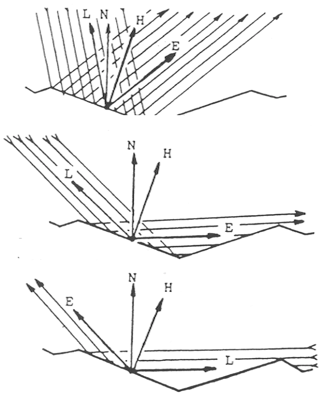
Self-shadowing a major effect



Self-Shadowing: V-Groove Model

Assumptions (Torrance-Sparrow)

1. **Symmetric, longitudinal, isotropically-distributed**
2. **Upper edges lie in plane**



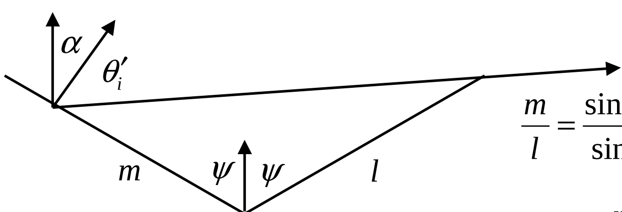
$$G = \min(G_a, G_b, G_c)$$

$$G_a = 1$$

$$G_b = \frac{2(\mathbf{N}\bullet\mathbf{H})(\mathbf{N}\bullet\mathbf{E})}{(\mathbf{H}\bullet\mathbf{E})}$$

$$G_c = \frac{2(\mathbf{N}\bullet\mathbf{H})(\mathbf{N}\bullet\mathbf{L})}{(\mathbf{H}\bullet\mathbf{L})}$$

Self-Shadowing: V-Groove Model



$$\sin l = \cos\theta_i'$$
$$\cos l = \sin\theta_i'$$
$$\sin\psi = \cos\alpha$$
$$\cos\psi = \sin\alpha$$

$$\frac{m}{l} = \frac{\sin m}{\sin l}$$

$$\begin{aligned}
\sin m &= \sin l + 2\psi \\
&= \sin l \cos 2\psi + \cos l \sin 2\psi \\
&= \cos\theta_i' \cos 2\psi + \sin\theta_i' \sin 2\psi \\
&= \cos\theta_i'(1 - 2\sin^2\psi) + \sin\theta_i' 2\cos\psi\sin\psi \\
&= \cos\theta_i'(1 - 2\cos^2\alpha) + \sin\theta_i' 2\cos\alpha\sin\alpha \\
&= \cos\theta_i' - 2\cos\alpha\left(\cos\alpha\cos\theta_i' - \sin\alpha\sin\theta_i''\right) \\
&= \cos\theta_i' - 2\cos\alpha\cos\left(\alpha + \theta_i'\right) \\
&= \cos\theta_i' - 2\cos\alpha\cos\theta_r \\
&= \hat{\mathbf{H}}\bullet\hat{\mathbf{E}} - 2\left(\hat{\mathbf{N}}\bullet\hat{\mathbf{H}}\right)\left(\hat{\mathbf{N}}\bullet\hat{\mathbf{E}}\right)
\end{aligned}$$

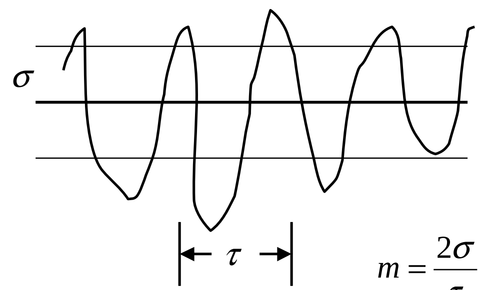
$$\begin{aligned}
G &= 1 - \frac{m}{l} \\
&= 1 - \frac{\sin m}{\sin l} \\
&= \frac{\hat{\mathbf{H}}\bullet\hat{\mathbf{E}} - \hat{\mathbf{H}}\bullet\hat{\mathbf{E}} + 2(\hat{\mathbf{N}}\bullet\hat{\mathbf{H}})(\hat{\mathbf{N}}\bullet\hat{\mathbf{E}})}{\hat{\mathbf{H}}\bullet\hat{\mathbf{E}}} \\
&= \frac{2(\hat{\mathbf{N}}\bullet\hat{\mathbf{H}})(\hat{\mathbf{N}}\bullet\hat{\mathbf{E}})}{\hat{\mathbf{H}}\bullet\hat{\mathbf{E}}}
\end{aligned}$$

Gaussian Rough Surface

Beckmann

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$

$$D(\alpha) = \frac{1}{\sqrt{\pi} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}}$$



$$m = \frac{2\sigma}{\tau}$$

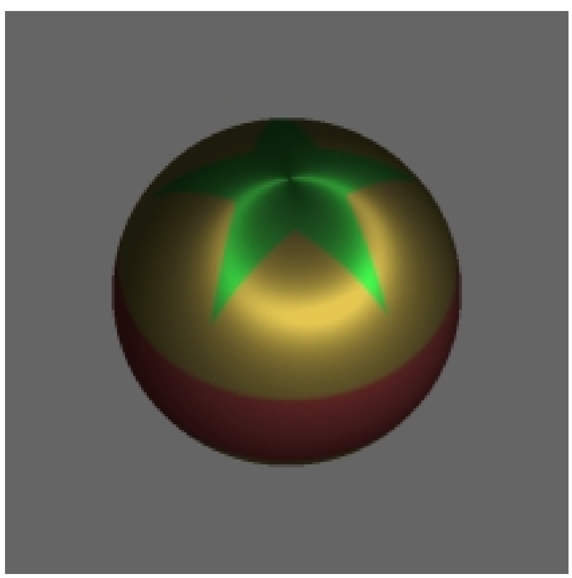
Smith

Derives shadowing function probabilistically

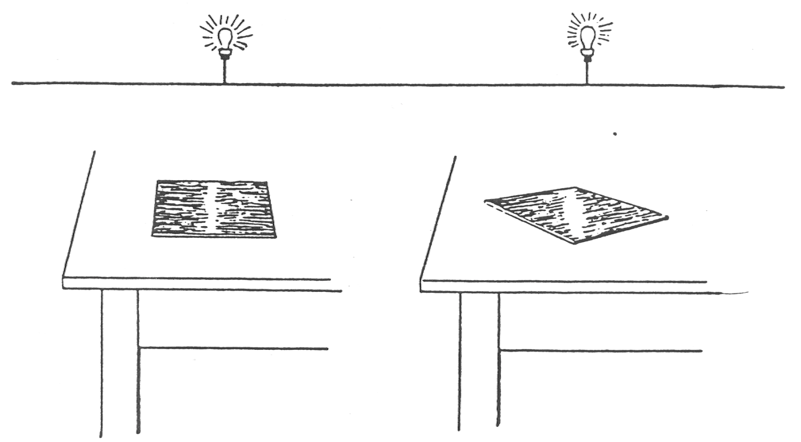
Self-consistency condition

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

Anisotropic Reflection



Anisotropic Reflection



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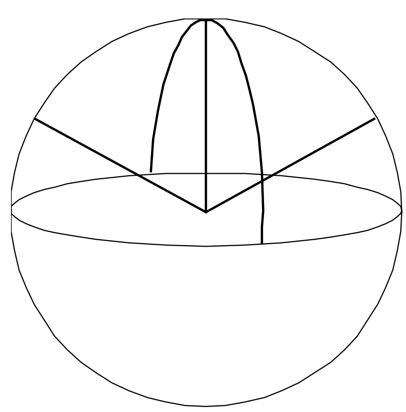
Anisotropic Reflection



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Anisotropic Reflection