Image formation

CS 178, Spring 2011

Begun 3/29/11. Finished 3/31/11.

Marc Levoy
Computer Science Department
Stanford University

Outline

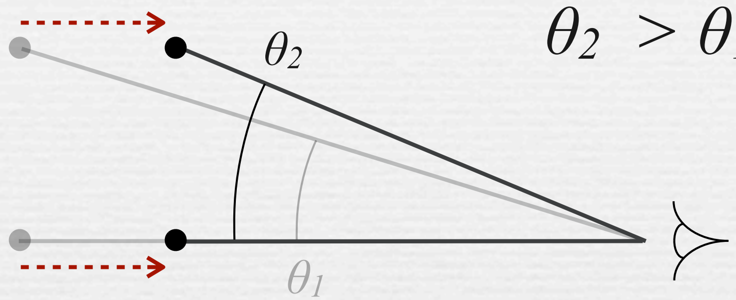
- perspective
  - natural versus linear perspective
  - vanishing points
- image formation
  - pinhole cameras
  - lenses
- exposure
  - shutter speed
  - aperture
  - ISO
- choosing a camera

The laws of perspective

- common assumptions
  1. Light leaving an object travels in straight lines.
  2. These lines converge to a point at the eye.

- natural perspective (Euclid, 3rd c. B.C.)
  3a. More distant objects subtend smaller visual angles.

The laws of perspective



$\theta_2 > \theta_1$

- natural perspective (Euclid, 3rd c. B.C.)
  3a. More distant objects subtend smaller visual angles.

Roman wall paintings

from Villa Publius Fannius Synistor,
Boscoreale, Pompeii (c. 40 B.C.)

Still life with peaches, from
Herculaneum (before 79 A.D.)

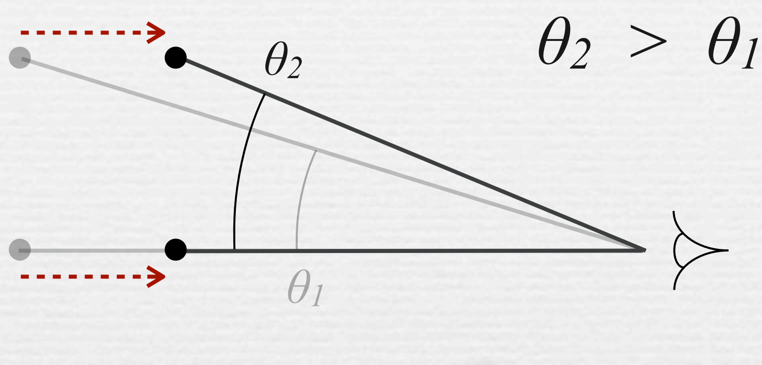
The laws of perspective

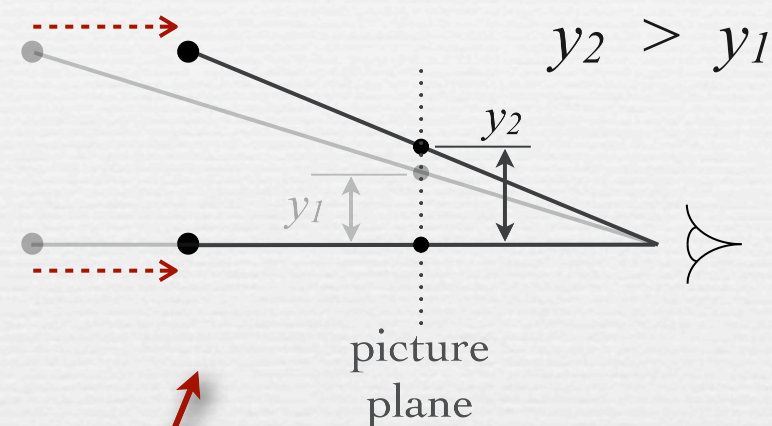
- common assumptions
  1. Light leaving an object travels in straight lines.
  2. These lines converge to a point at the eye.

- natural perspective (Euclid, 3rd c. B.C.)
  3a. More distant objects subtend smaller visual angles.

- linear perspective (Filippo Brunelleschi, 1413)
  3b. A perspective image is formed by the intersection of these lines with a "picture plane" (the canvas).

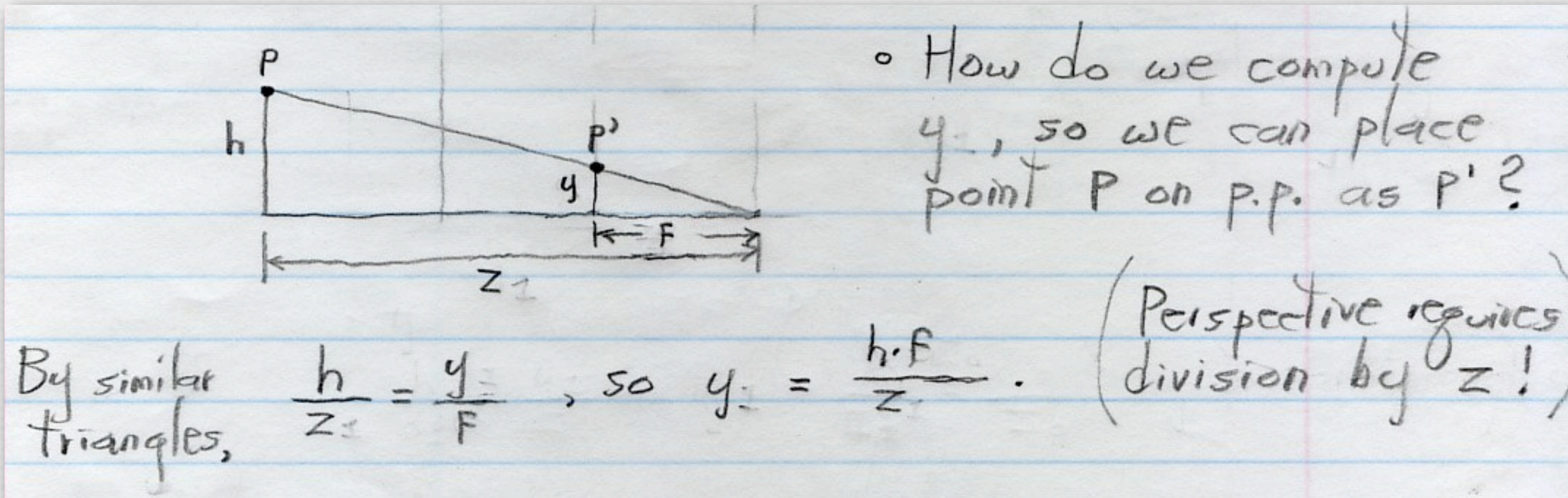
The laws of perspective





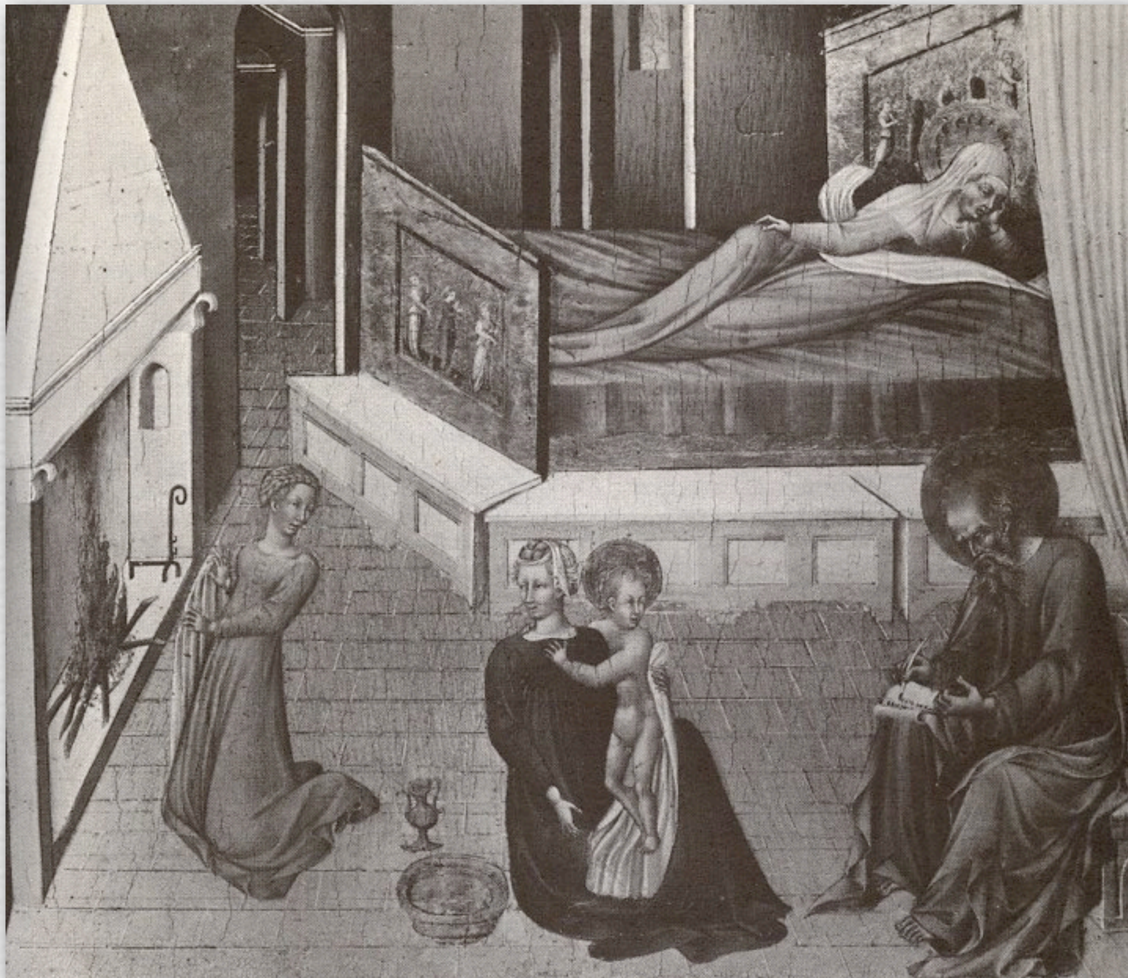
- natural perspective (Euclid, 3rd c. B.C.)
  - 3a. More distant objects subtend smaller visual angles.
- linear perspective (Filippo Brunelleschi, 1413)
  - 3b. A perspective image is formed by the intersection of these lines with a "picture plane" (the canvas).

Projection onto picture plane (contents of whiteboard)



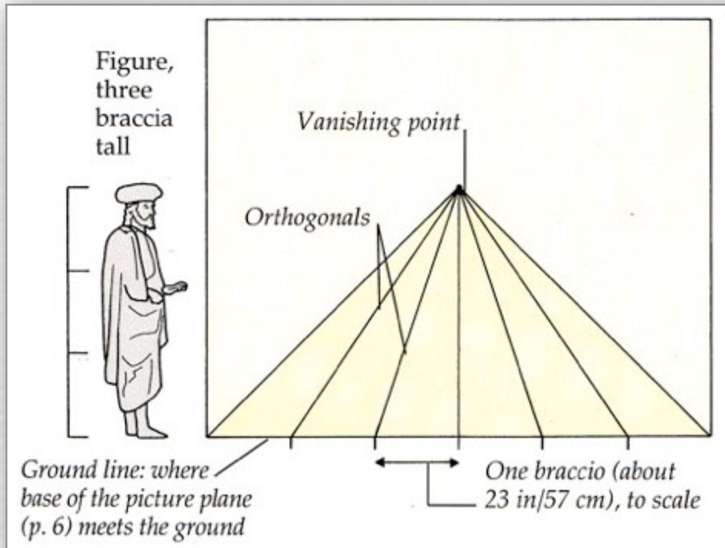
Filippo Brunelleschi,
dome of the cathedral,
Florence (1419)

The problem of drawing pavimento

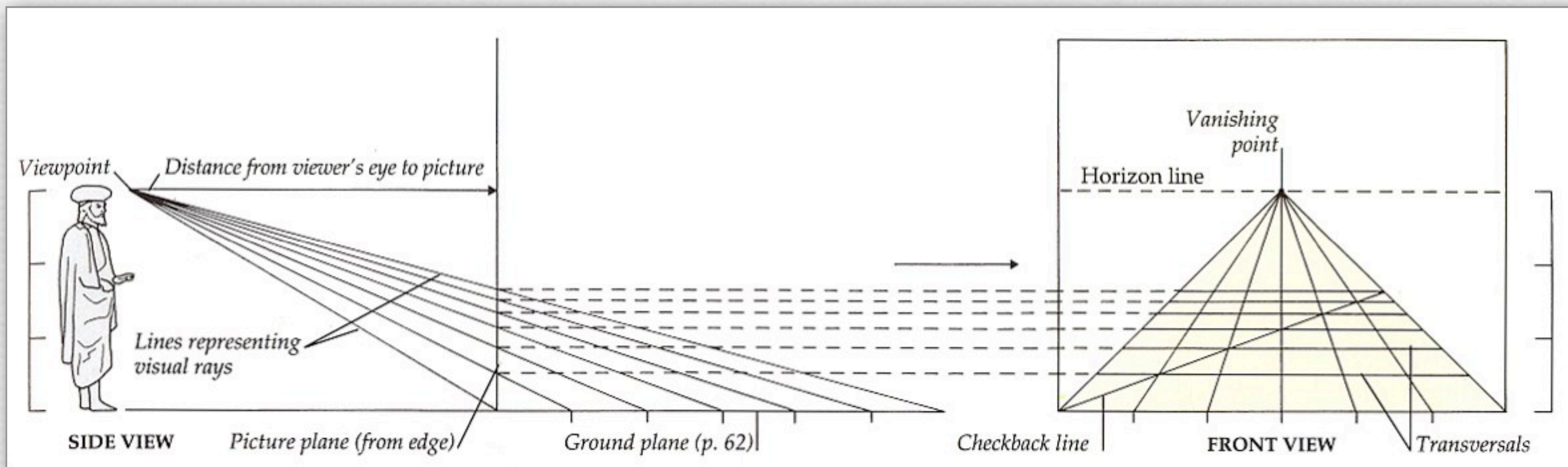


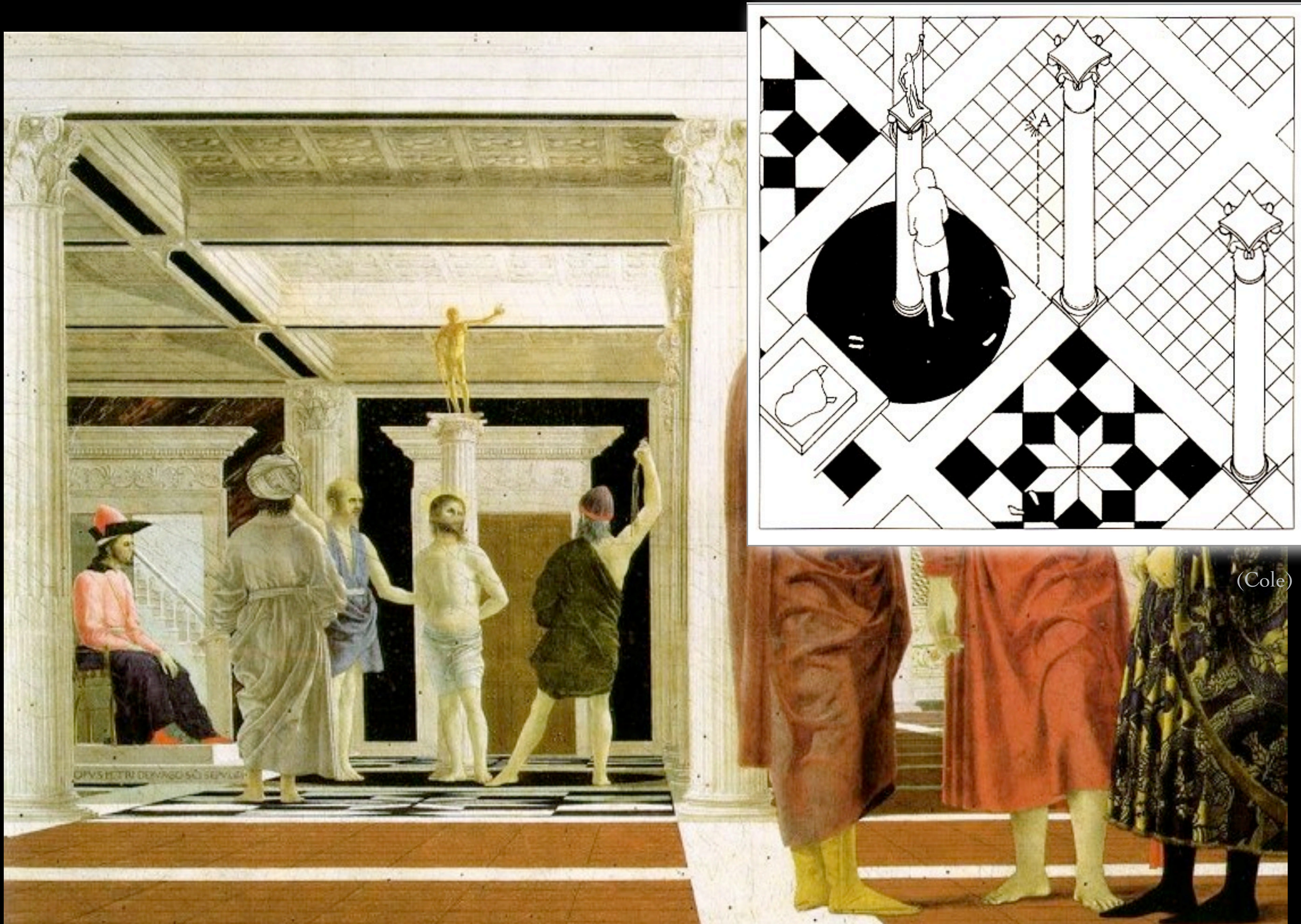
Giovanni de Paolo, Birth of St. John the Baptist (1420)

Alberti's method (1435)



(Cole)



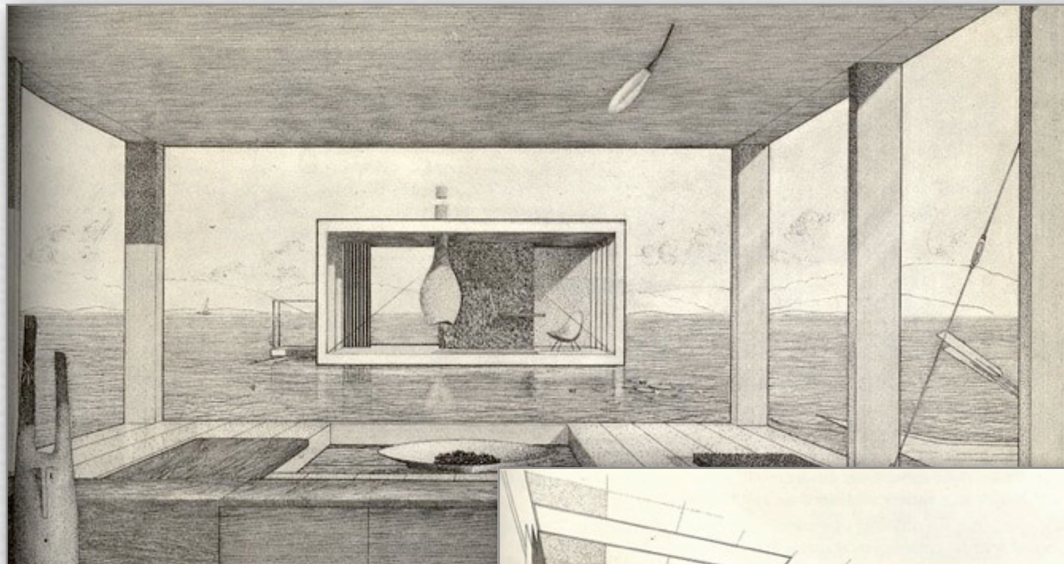


(Cole)

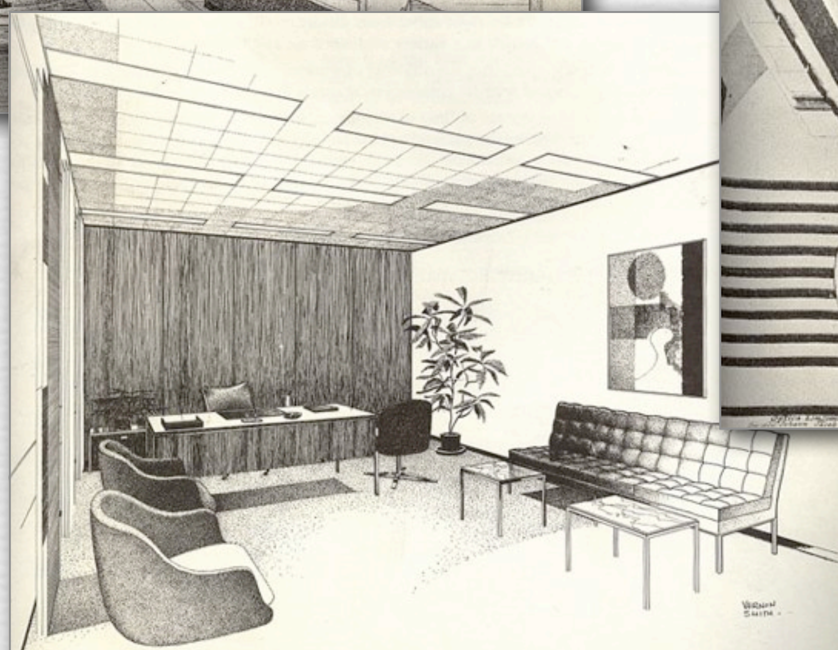
Piero della Francesca, The Flagellation (c.1460)

Vanishing points

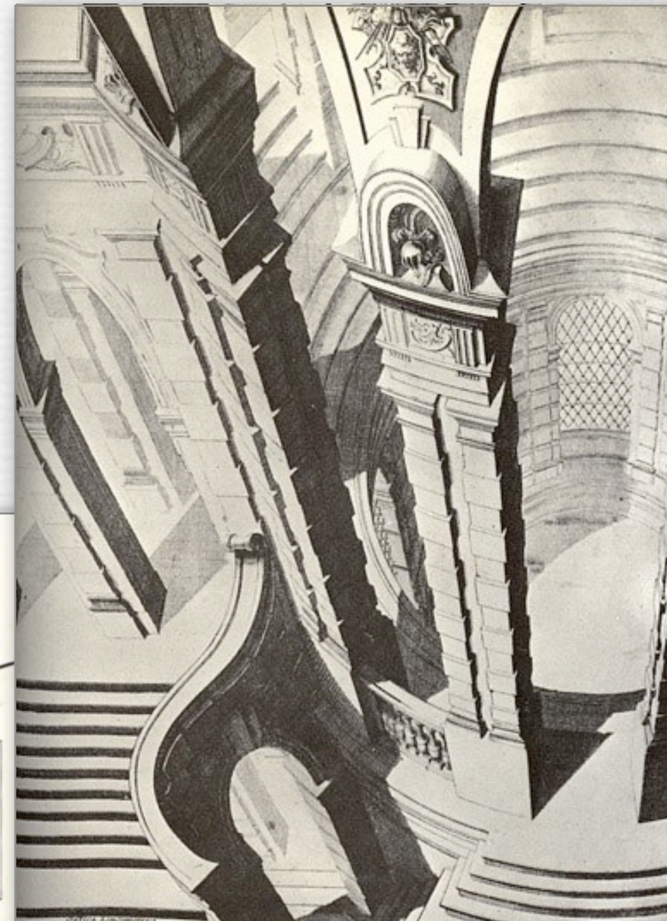
Q. How many vanishing points can there be in a perspective drawing?



1-point



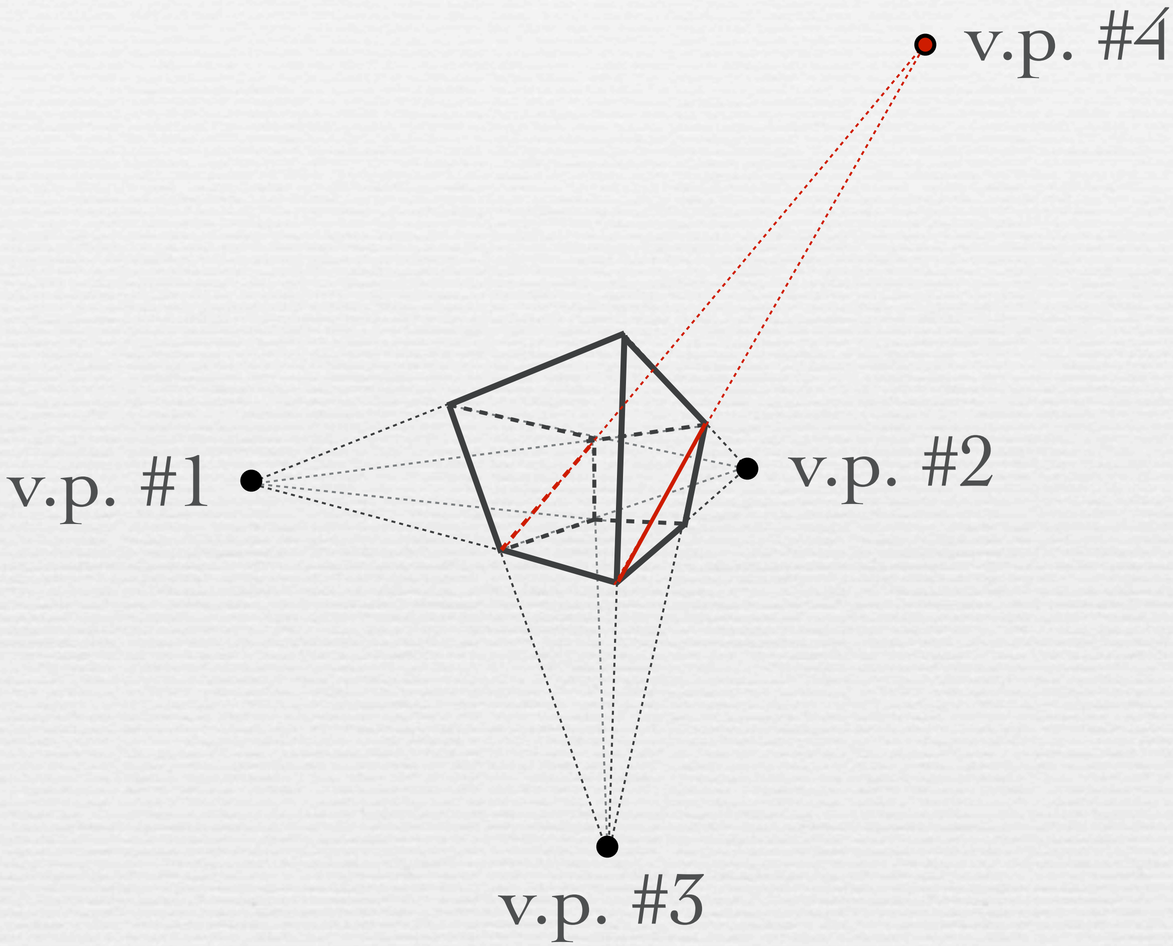
2-point



3-point

(D'Amelio)

Example of a 4th vanishing point



- each direction of parallel lines will converge to a unique vanishing point

Q. Should the distant ends of a long facade be drawn smaller than its center in a perspective drawing?

?

- no, in linear perspective straight lines remain straight
- lines parallel to the picture plane do not converge
- they appear smaller when you view the drawing, due to natural perspective (angles subtended at eye)

Recap

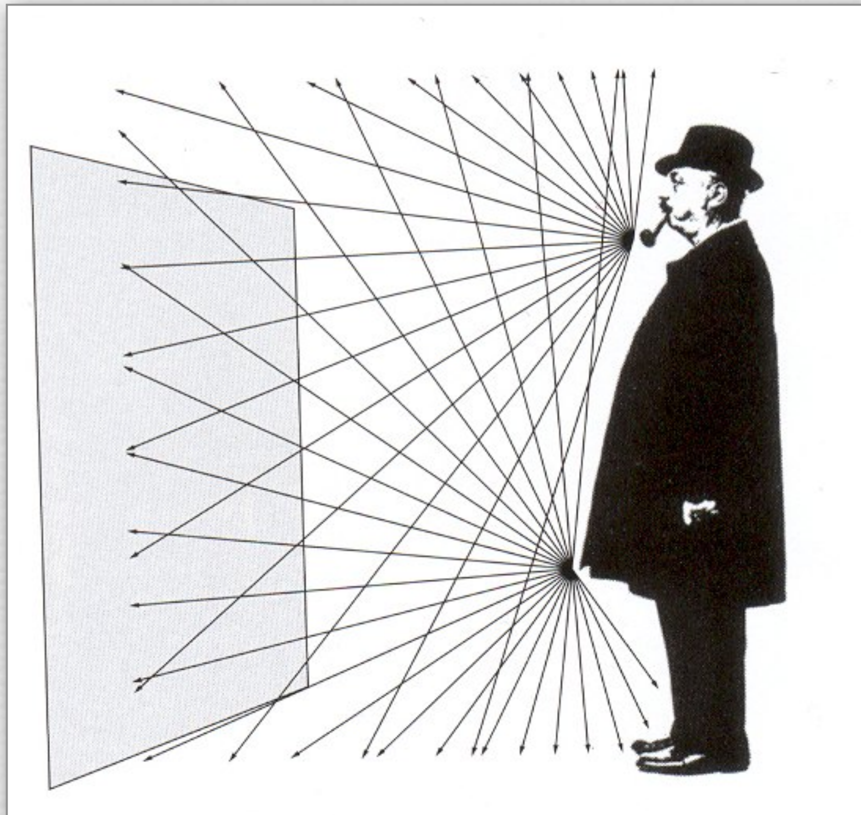
- natural perspective
  - visual angle subtended by a feature in the world
- linear perspective
  - intersections of lines of sight with a picture plane
  - the correct way to make a drawing on a flat surface
- vanishing points
  - one per direction of line in the scene
  - lines parallel to the picture plane do not converge

Questions?

Single lens reflex camera (SLR)

Nikon F4
(film camera)

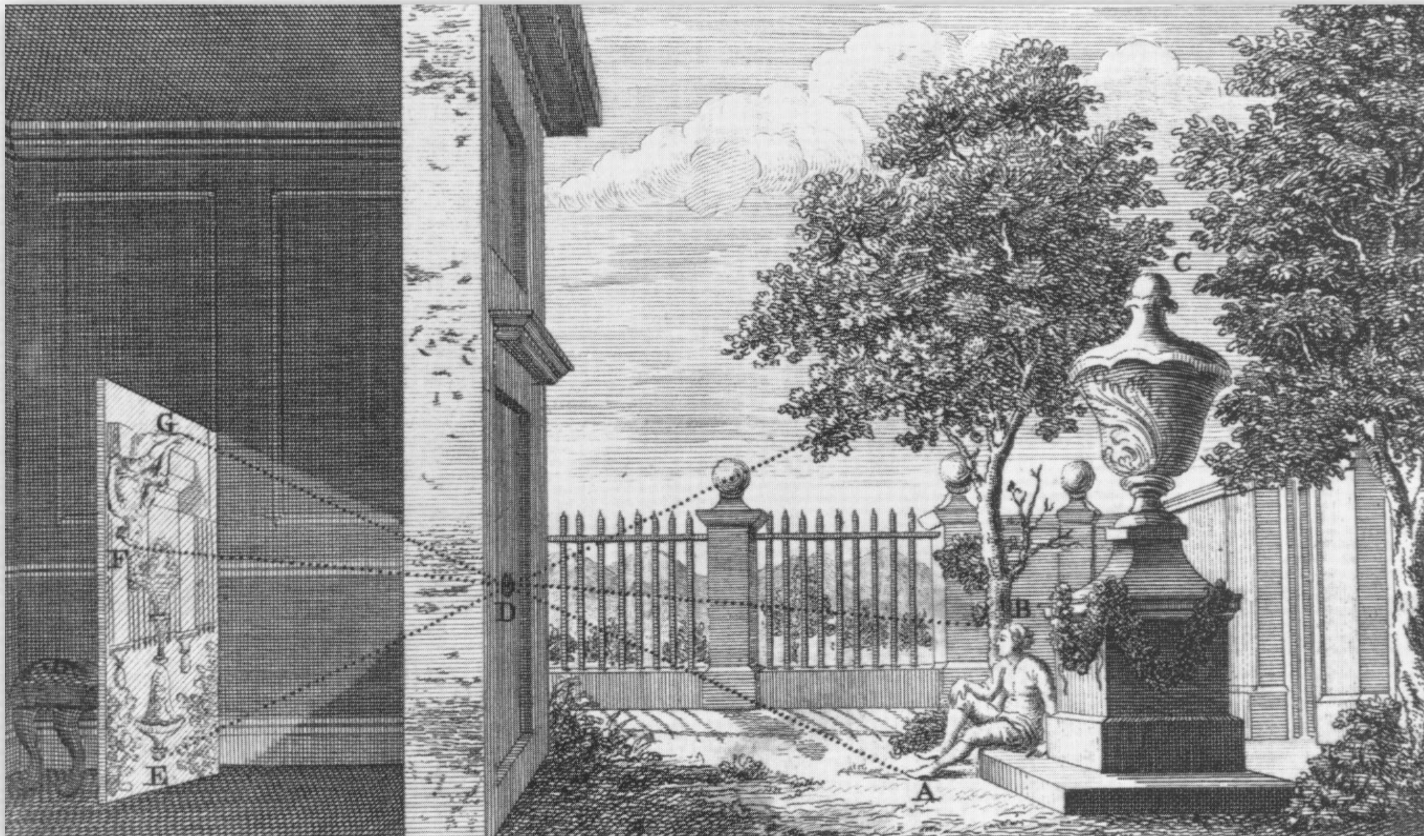
Why not use sensors without optics?



(London)

- each point on sensor would record the integral of light arriving from every point on subject
- all sensor points would record similar colors

Pinhole camera
(a.k.a. *camera obscura*)

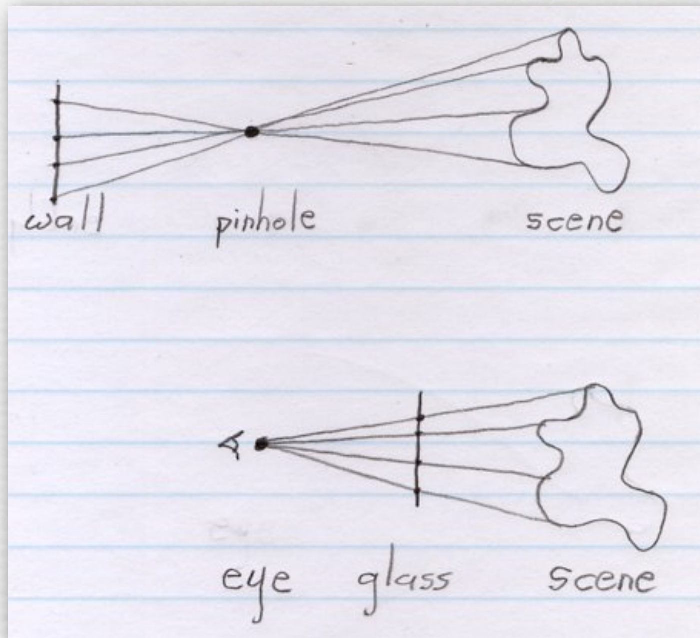


- linear perspective with viewpoint at pinhole
- tilting the picture plane changes the number and location of vanishing points

Equivalence of Dürer's glass and *camera obscura* (contents of whiteboard)

camera obscura

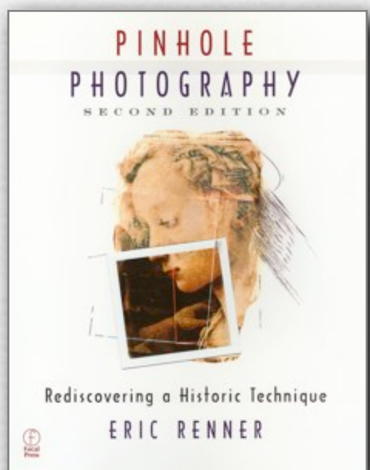
Dürer's glass



- both devices compute <u>2D planar geometric projections</u>, i.e. projections along straight lines through a point and onto a plane
  - the images differ only in scale (and a reflection around the origin)

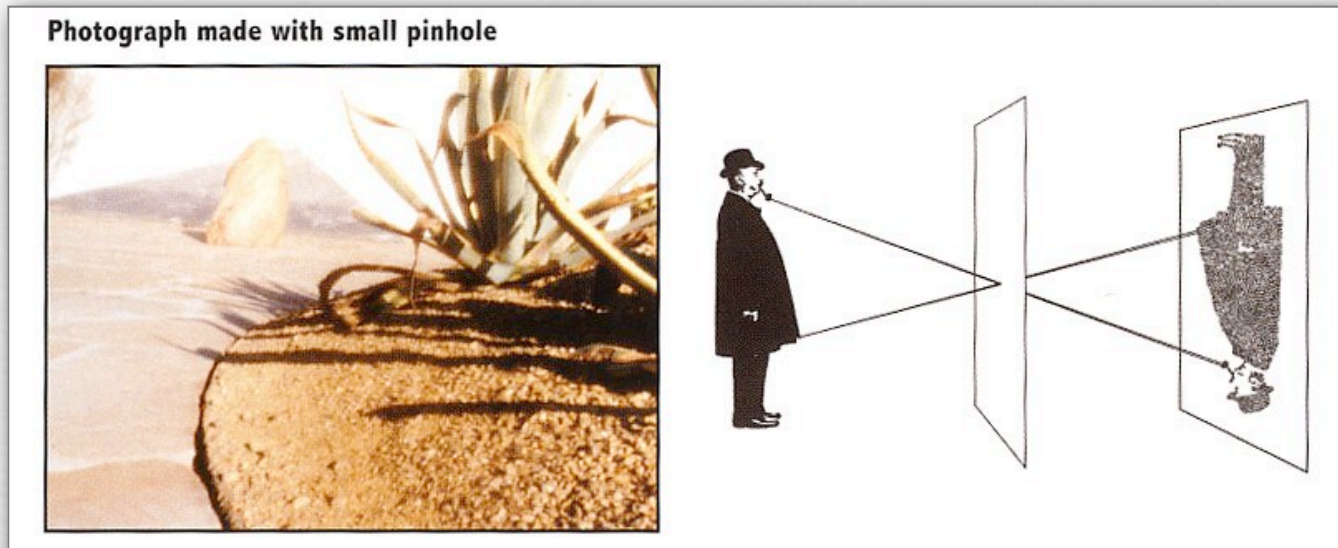
Pinhole photography

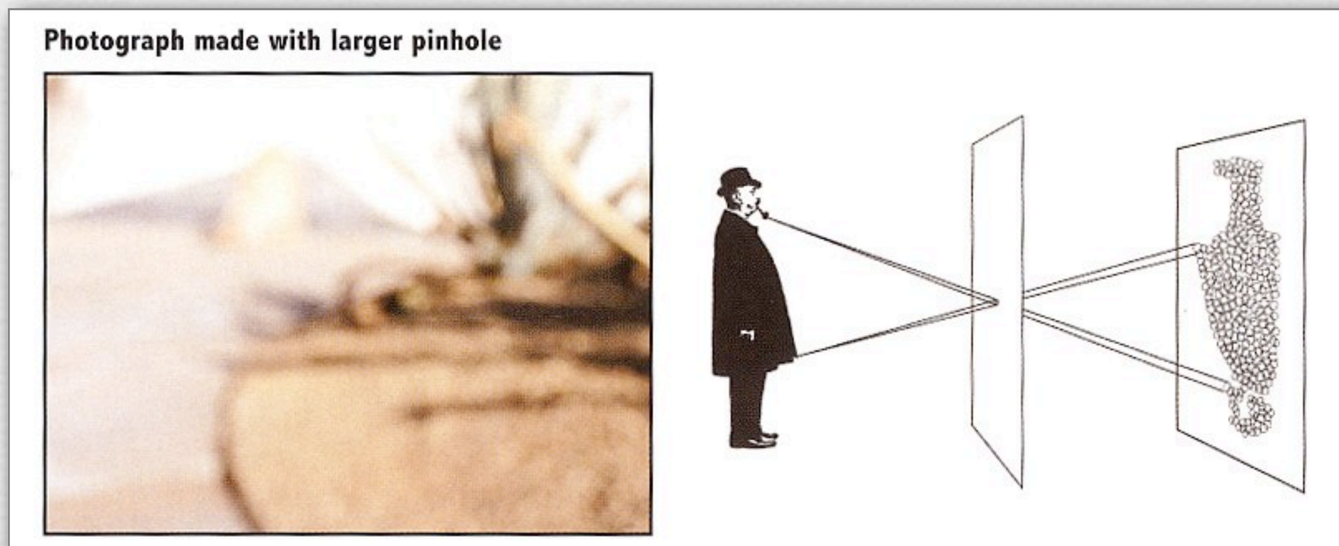
- no distortion
  - straight lines remain straight
- infinite depth of field
  - everything is in focus



(Bami Adedoyin)

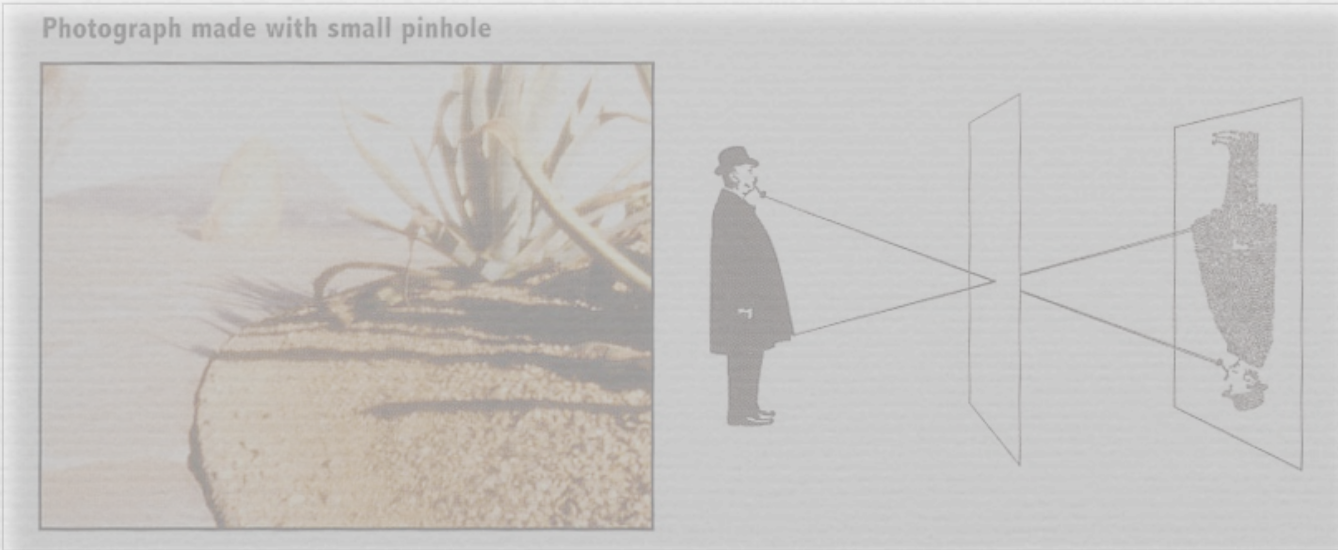
Effect of pinhole size

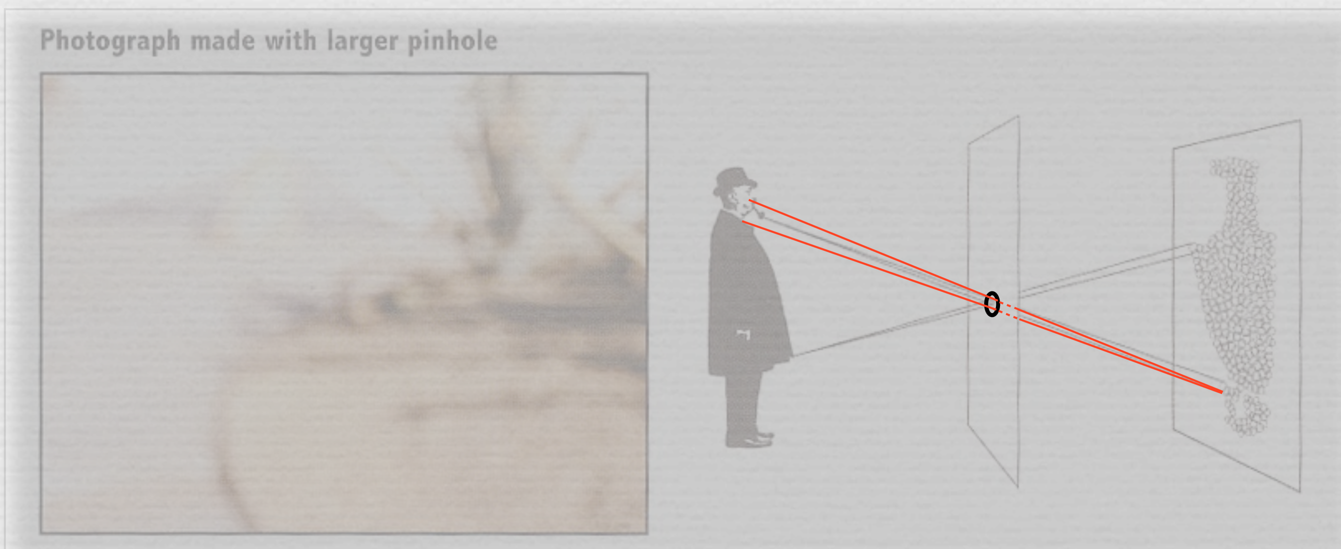




(London)

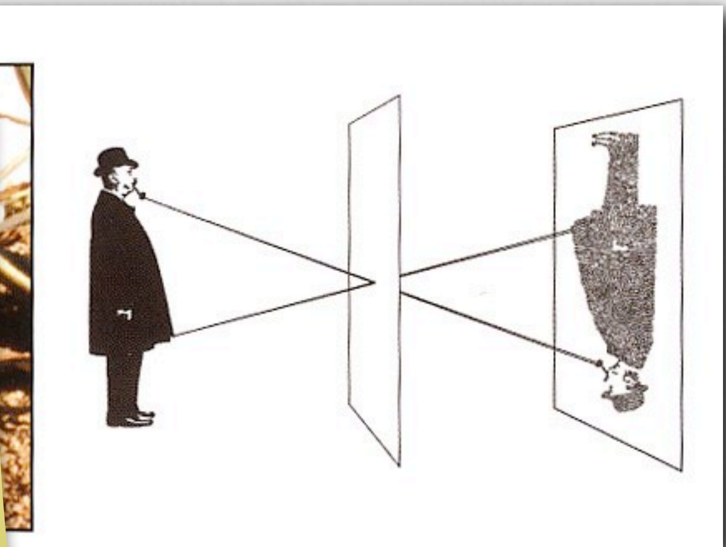
Effect of pinhole size





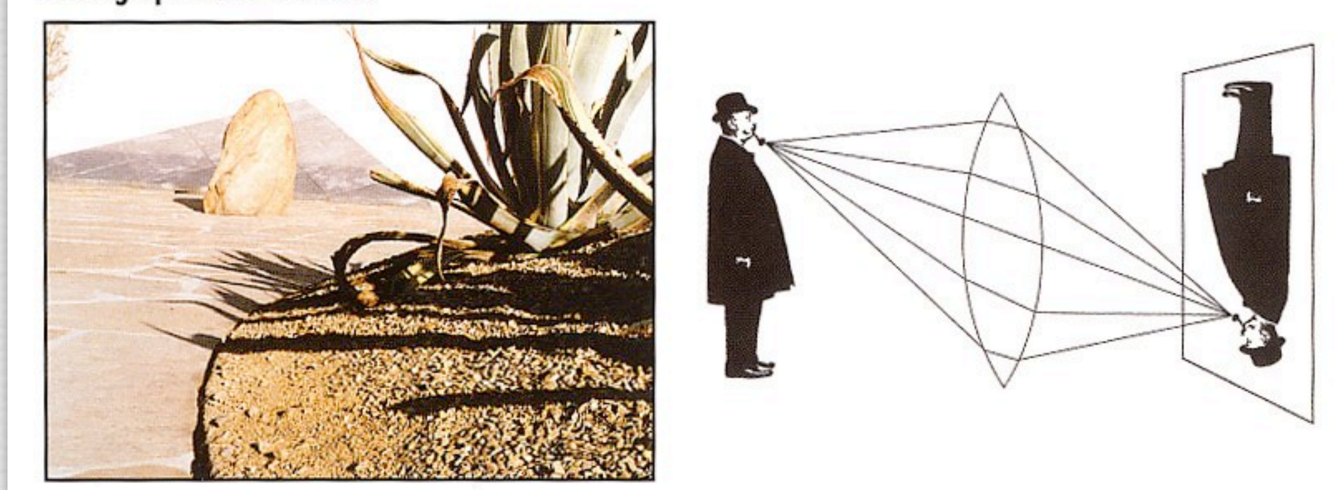
(London)

Replacing the pinhole with a lens



As I mentioned in class (but may not have made sufficiently clear), a photographic camera produces the same linear perspective projection as a camera obscura. In the photographic case, a lens replaces the pinhole of the camera obscura, and film or a digital sensor replaces the wall that receives the image in the camera obscura. The advantage of a lens over a pinhole is that it lets in more light, yet still makes a sharp image. The precise location of the "equivalent pinhole" inside a photographic lens is a topic we will consider next week, when we look at lens geometry.

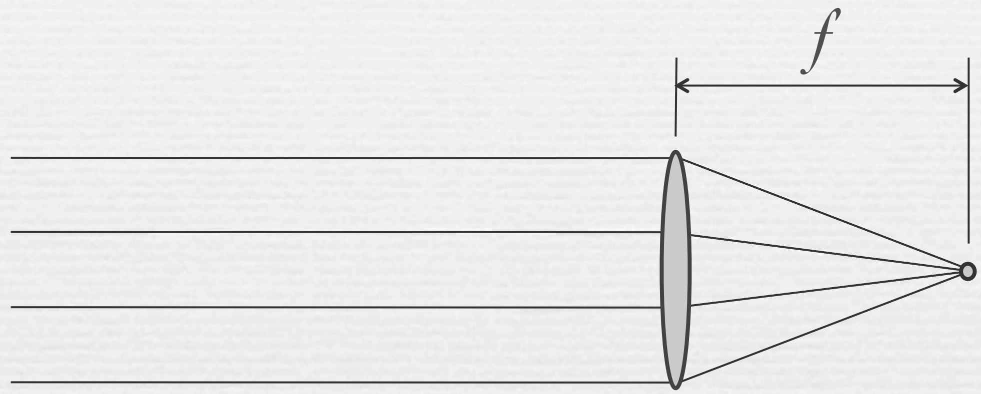
One of the implications of this equivalence is that rotating a camera, which tilts its sensor relative to the world, has the same effect as tilting the wall in a cameara obscura (or the glass plane in Durer's perspective glass) - it adds or removes vanishing points from the resulting perspective image.



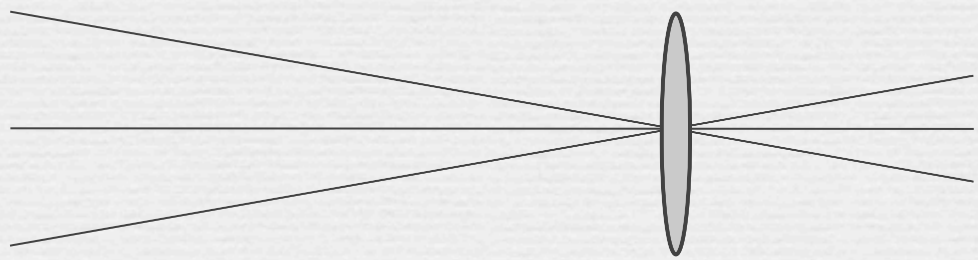
(London)

Geometrical optics

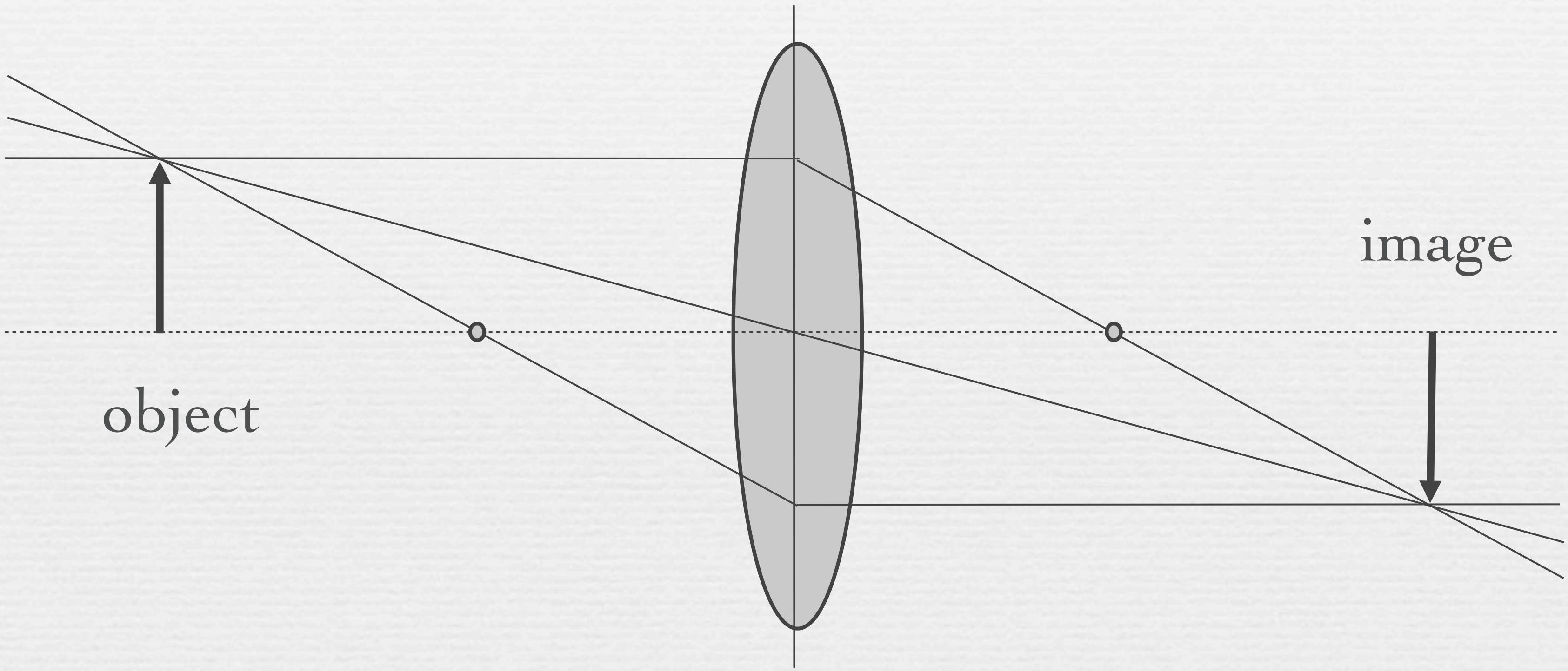
- parallel rays converge to a point located at focal length f from lens



- rays going through center of lens are not deviated
  - hence same perspective as pinhole



Gauss' ray tracing construction



- rays coming from points on a plane parallel to the lens are focused on another plane parallel to the lens

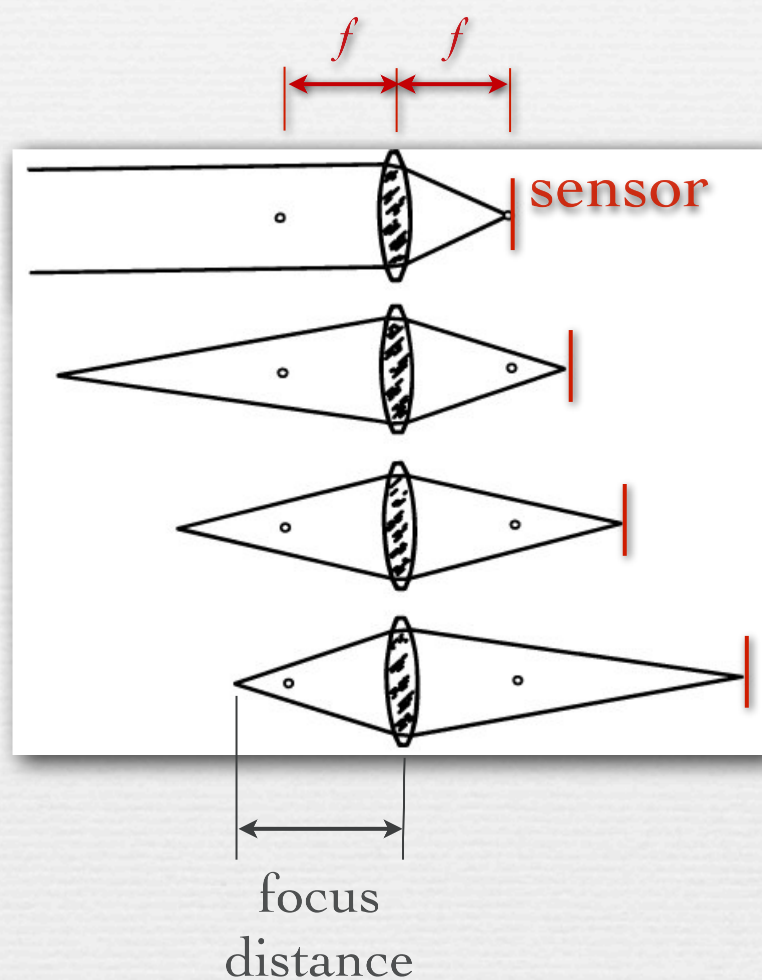
Changing the focus distance

- to focus on objects at different distances, move sensor relative to lens

Gauss's ray tracing construction isn't sufficient to understand why focus distance (i.e. distance between the lens and scene) should decrease as the distance between the lens and sensor increases. We'll investigate this relationship next week, and derive some formulas about it.

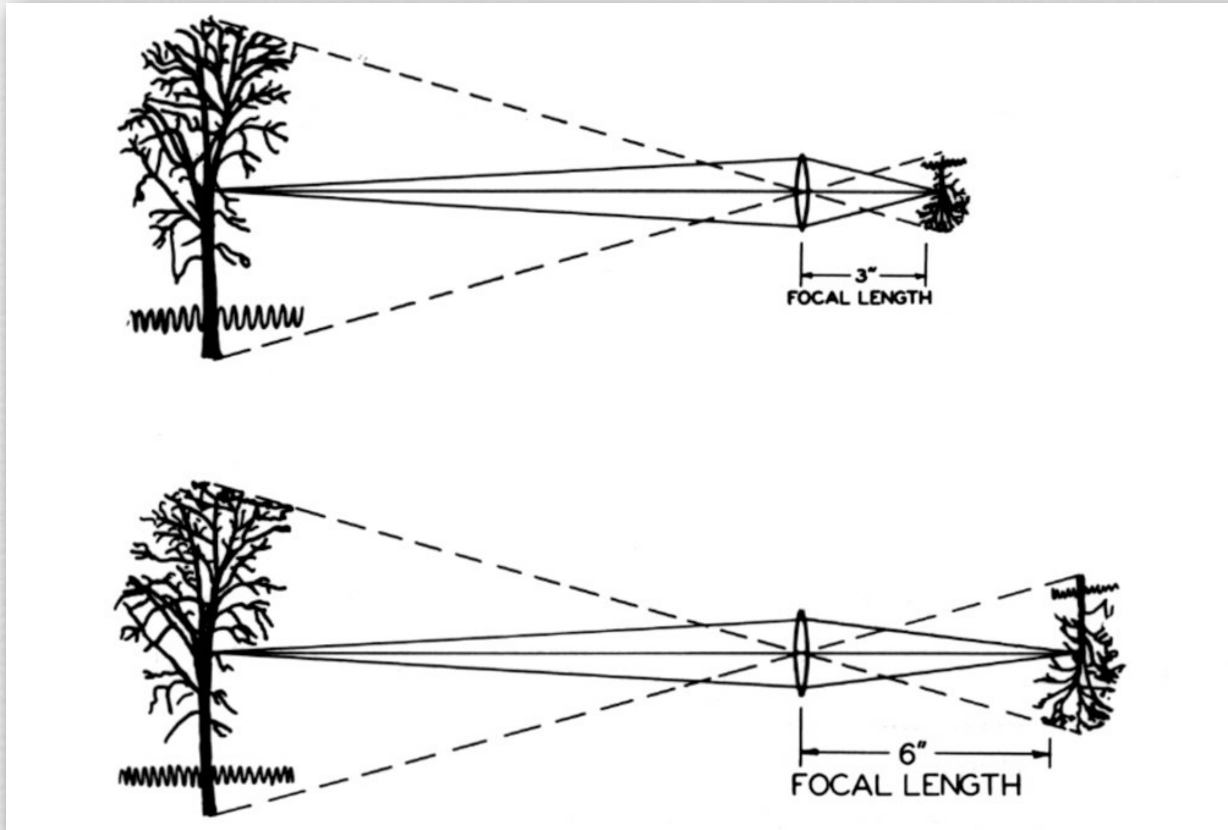
Also, although I've drawn the sensor as moving in this example, in practice one moves the lens - by focusing it. The sensor is fixed inside the camera body, and doesn't move.

by convention, the "focus distance" is on the object side of the lens



Changing the focal length

- weaker lenses have longer focal lengths
- to stay in focus, move the sensor further back
- focused image of tree is located slightly beyond the focal length

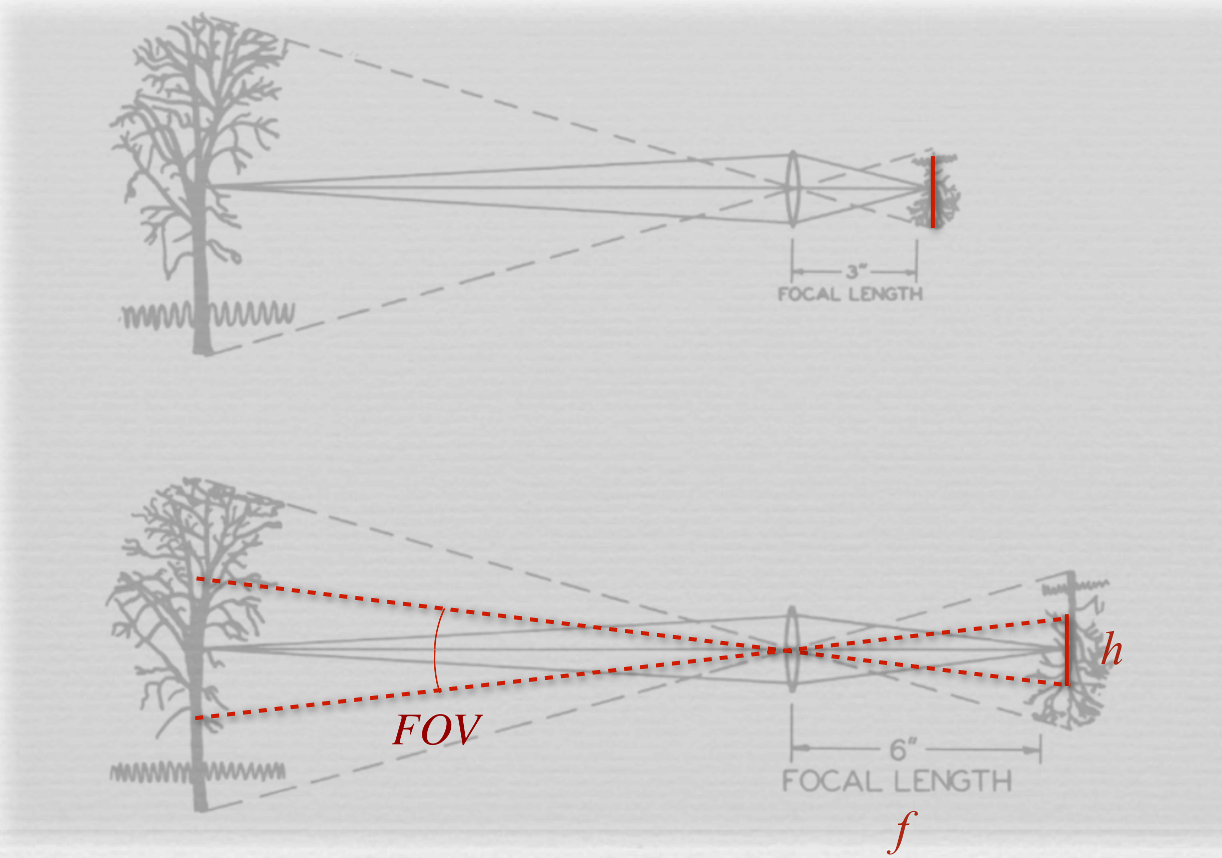


(Kingslake)

the tree would be in focus at the lens focal length only if it were infinitely far away

Changing the focal length

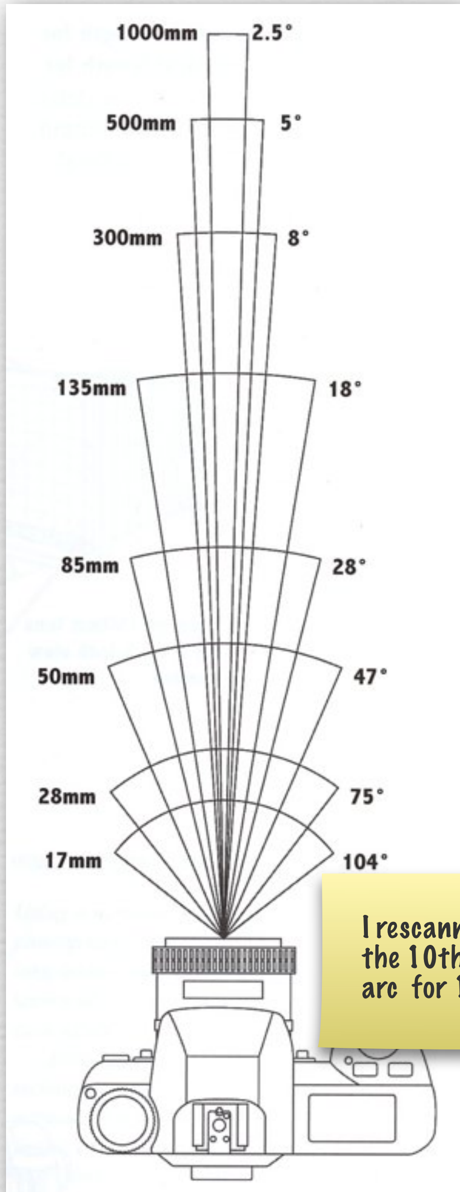
- if the sensor size is constant, the field of view becomes smaller



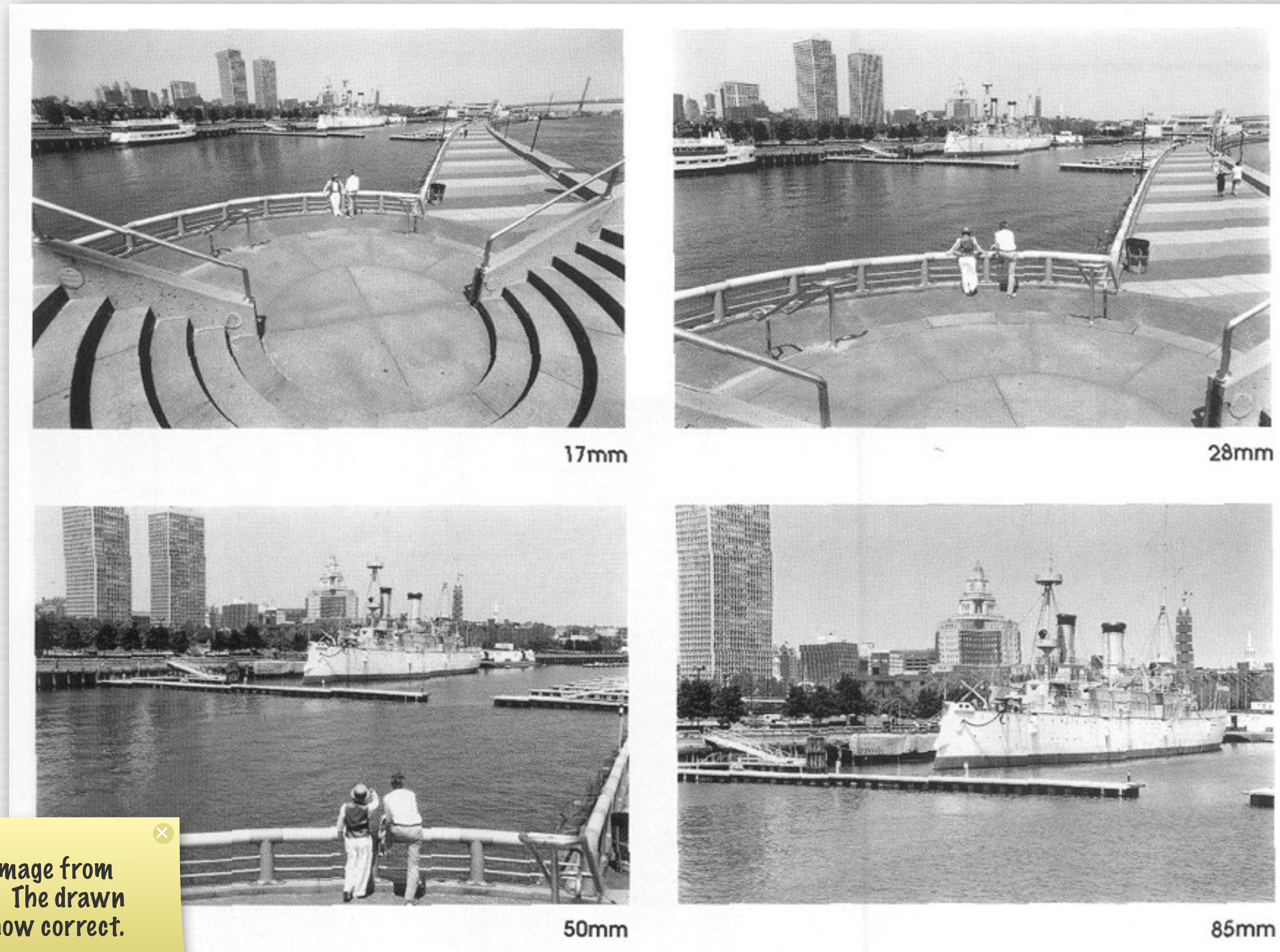
(Kingslake)

$$FOV = 2 \arctan (h / 2f)$$

Focal length and field of view



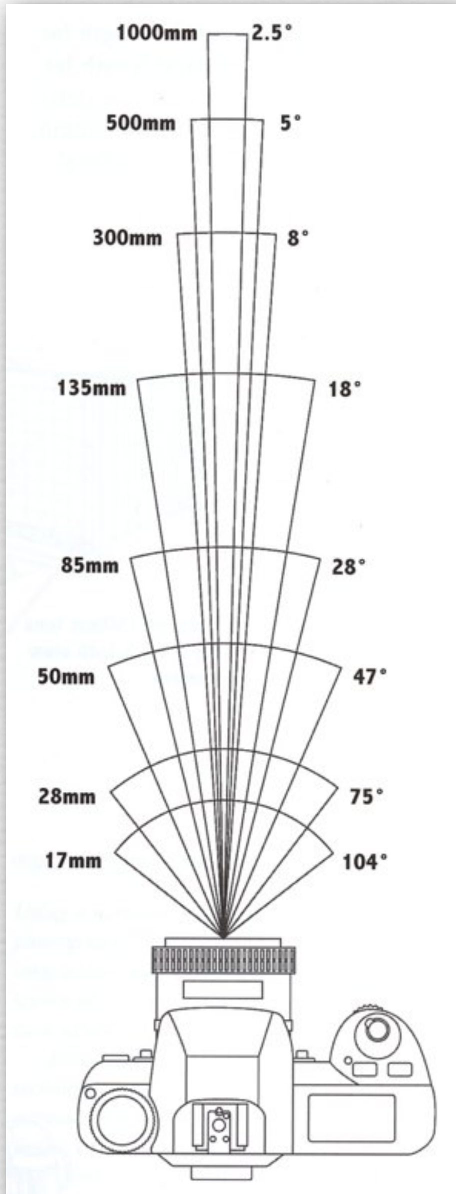
I rescanned this image from the 10th edition. The drawn arc for 104° is now correct.

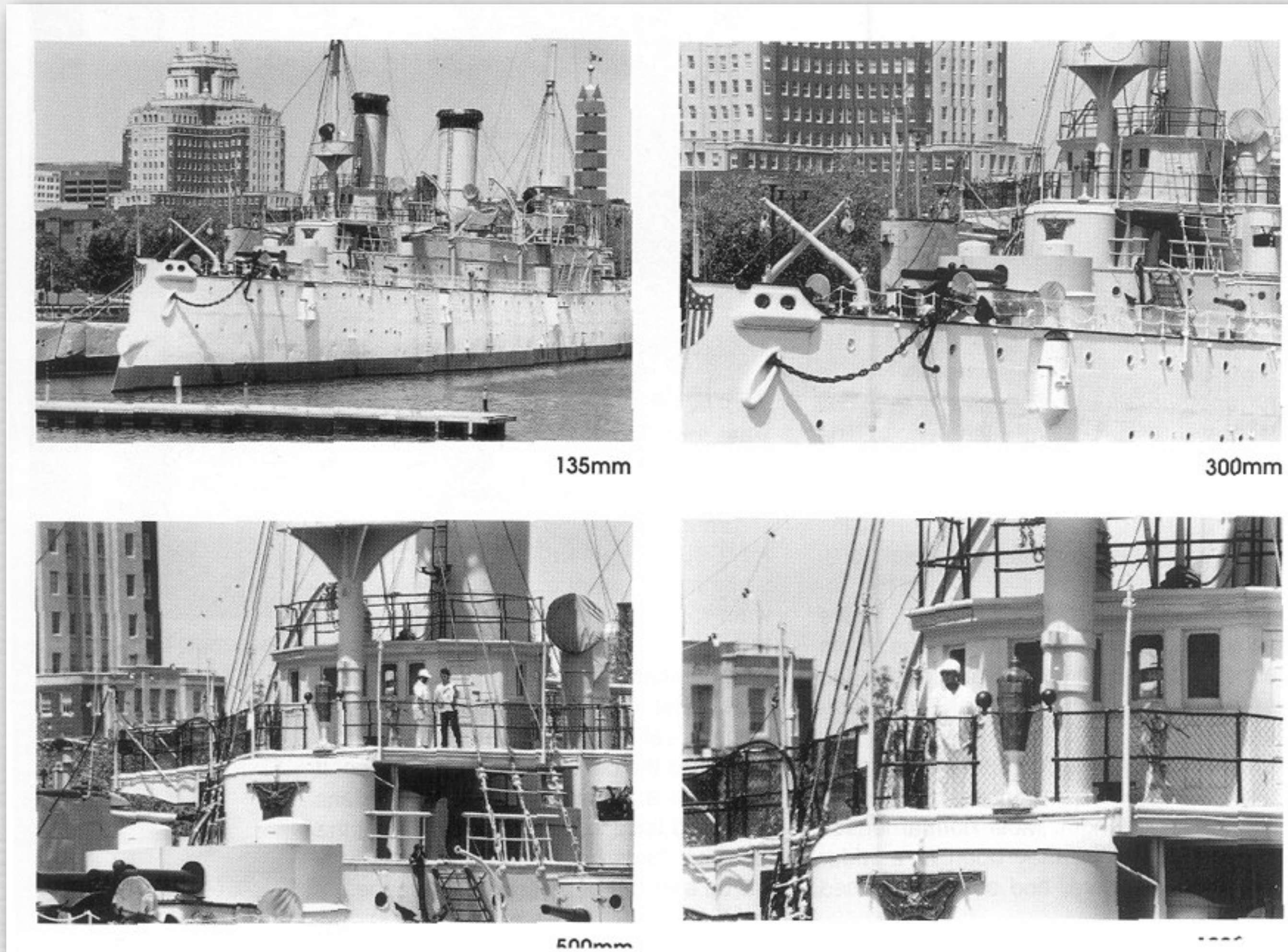


(London)

FOV measured diagonally on a 35mm full-frame camera (24 × 36mm)

Focal length and field of view



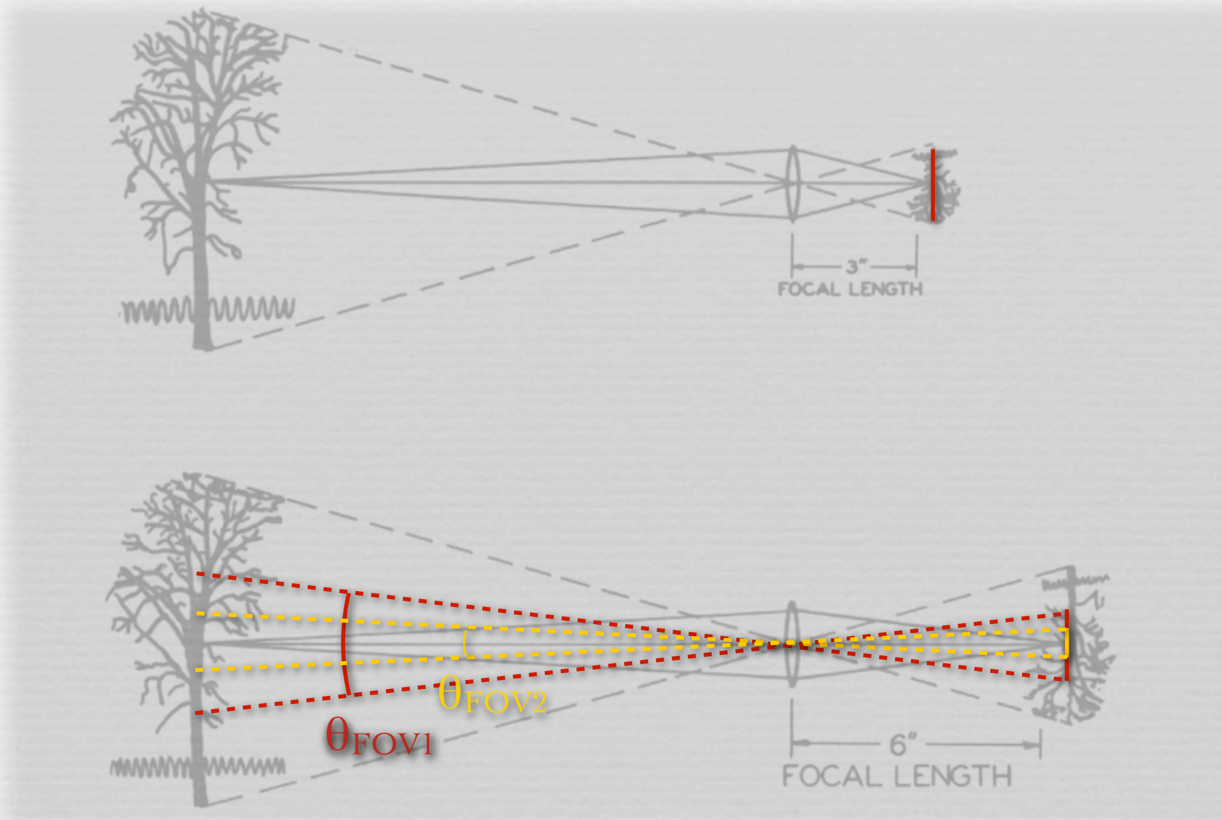


(London)

FOV measured diagonally on a
35mm full-frame camera (24 × 36mm)

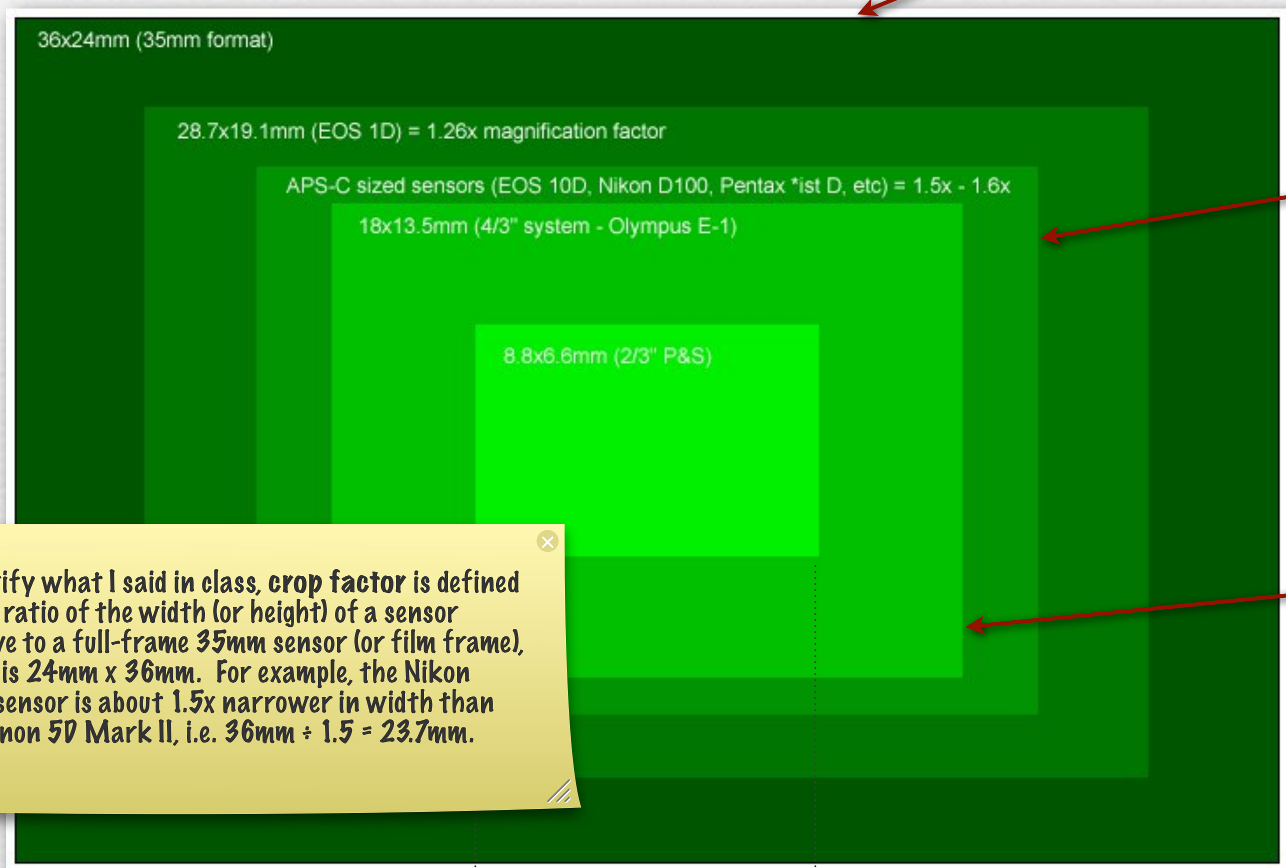
Changing the sensor size

- if the sensor size is smaller, the field of view is smaller too
- smaller sensors either have fewer pixels, or noiser pixels



(Kingslake)

Sensor sizes



To clarify what I said in class, **crop factor** is defined as the ratio of the width (or height) of a sensor relative to a full-frame 35mm sensor (or film frame), which is 24mm x 36mm. For example, the Nikon D40's sensor is about 1.5x narrower in width than the Canon 5D Mark II, i.e. 36mm ÷ 1.5 = 23.7mm.

"full frame"
Canon 5D Mark II
(24mm × 36mm)

"APS-C"
Nikon D40
(15.5mm × 23.7mm)
(~1.5× crop factor)

"micro 4/3"
Panasonic GF1
(13mm × 17.3mm)
(~2× crop factor)

"point-and-shoot"
Canon A590
(5.75mm × 4.31mm)
(~8× crop factor)

Paris, 2009 (Panasonic GF1 + Leica 90mm)

Changing the focal length versus changing the viewpoint

(Kingslake)

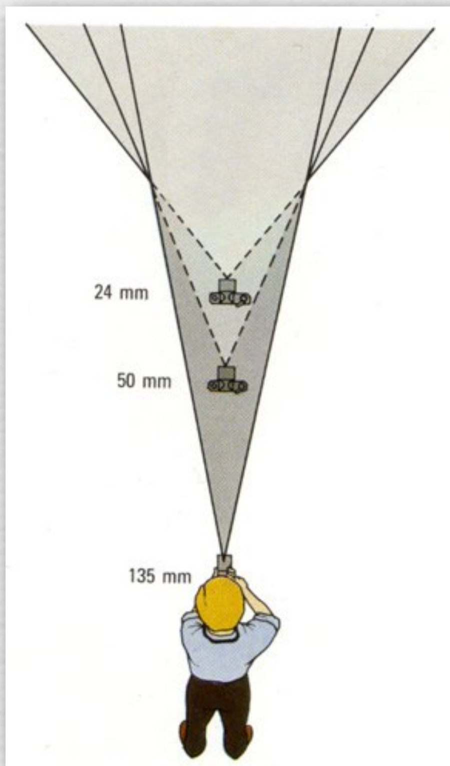
wide-angle

telephoto and moved back

- changing the focal length lets us move back from a subject, while maintaining its size on the image
- but moving back changes perspective relationships

Changing the focal length versus changing the viewpoint

- moving back while changing the focal length lets you keep objects at one depth the same size
- in cinematography, this is called the dolly zoom, or "Vertigo effect", after Alfred Hitchcock's movie



Effect of focal length on portraits

- standard "portrait lens" is 85mm

wide angle

standard

telephoto

Recap

- pinhole cameras compute correct linear perspectives
  - but dark
- lenses gather more light
  - but only one plane of scene is in focus
  - focus by moving the sensor or lens
- focal length determines field of view
  - from wide angle to telephoto
  - depends on sensor size

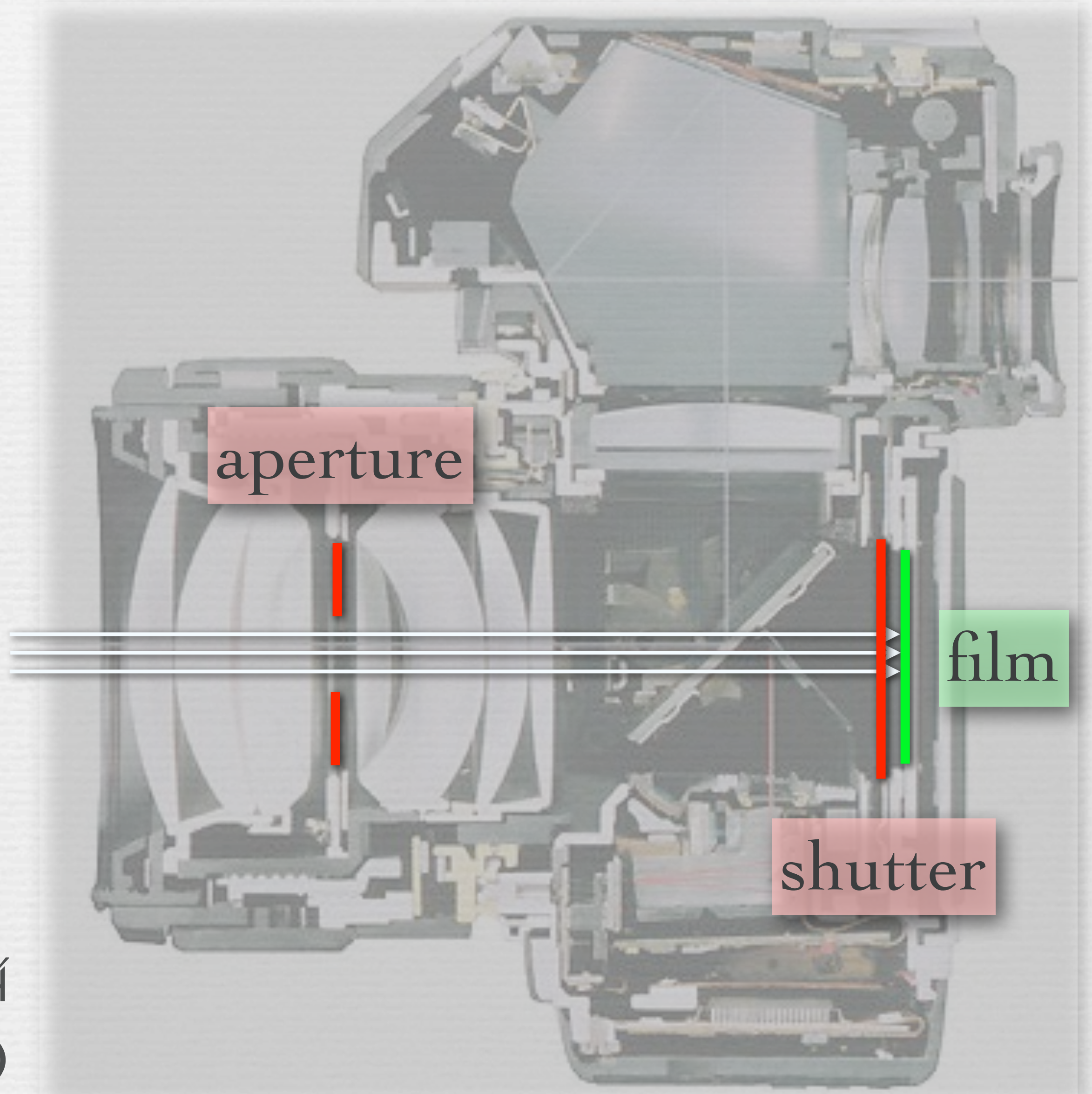
more in the lens lectures next week

Questions?

Exposure

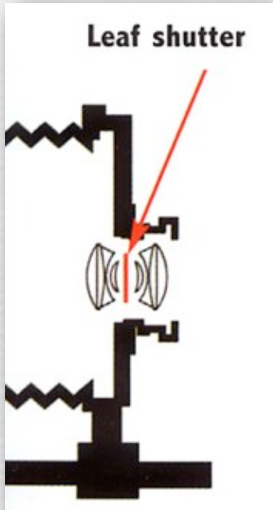
- $H = E \times T$
- exposure = irradiance × time
- irradiance (E)
  - amount of light falling on a unit area of sensor per second
  - controlled by aperture
- exposure time (T)
  - in seconds
  - controlled by shutter

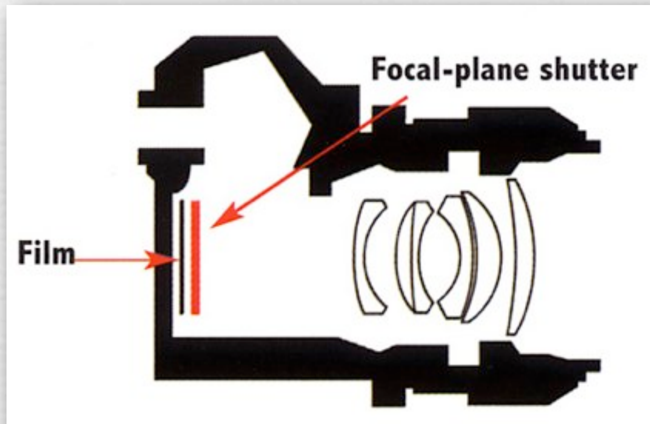
Single lens reflex camera



Nikon F4
(film camera)

Shutters





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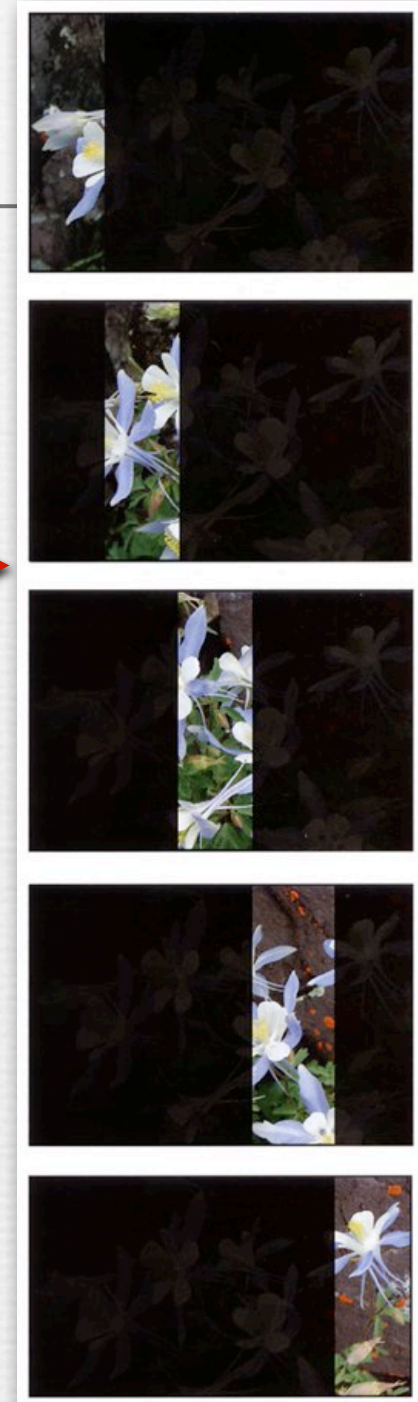


- quiet
- slow
  (max 1/500s)
- out of focus
- need one
  per lens

- loud
- fast
  (max 1/4000)
- in focus
- distorts motion

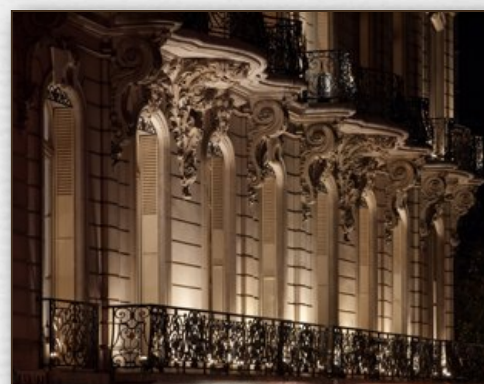
Jacques-Henri Lartigue, Grand Prix (1912)

Shutter speed

- controls how long the sensor is exposed to light
- linear effect on exposure until sensor saturates

- denoted in fractions of a second:
  - 1/2000, 1/1000,...,1/250, 1/125, 1/60,...,15, 30, B(ulb)

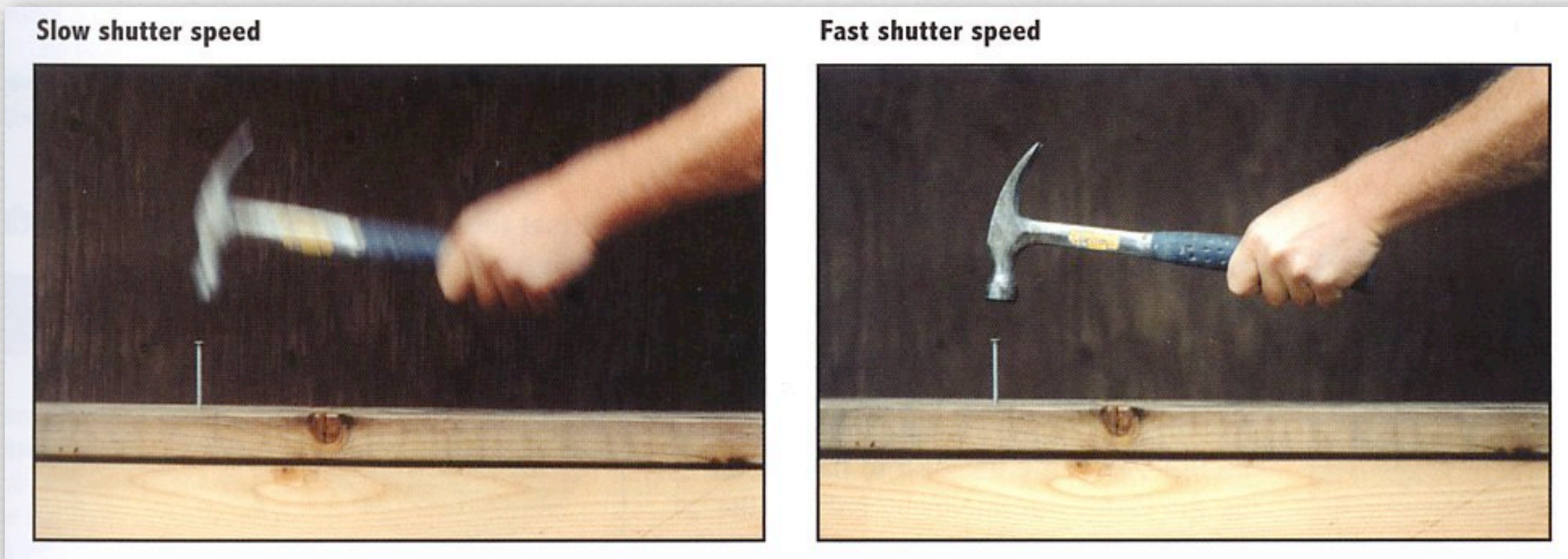
- normal humans can hand-hold down to 1/60 second
  - *rule of thumb:* shortest exposure = 1 / *f*
  - e.g. 1/180 second for a 180mm lens



GF1 (2× crop) + Leica 90mm

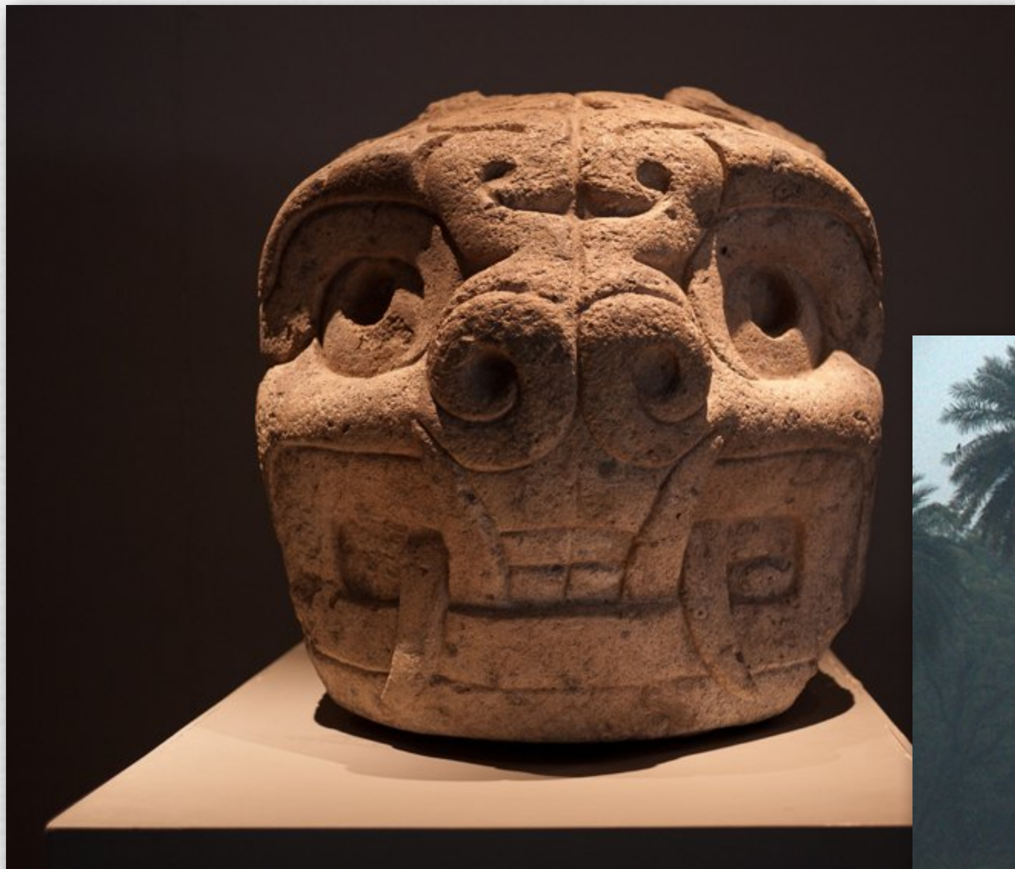
Main side-effect of shutter speed

- motion blur
- doubling exposure time doubles motion blur



(London)

Useful shutter speeds



1/25 sec (lucky!)

1/40 sec

Useful shutter speeds

1/125 sec

1/250 sec

Useful shutter speeds

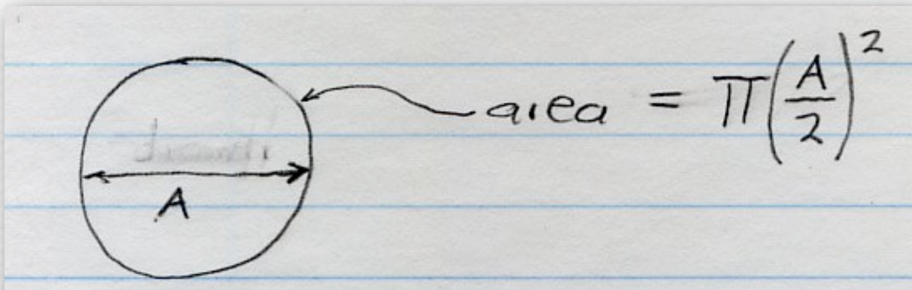
1/800 sec

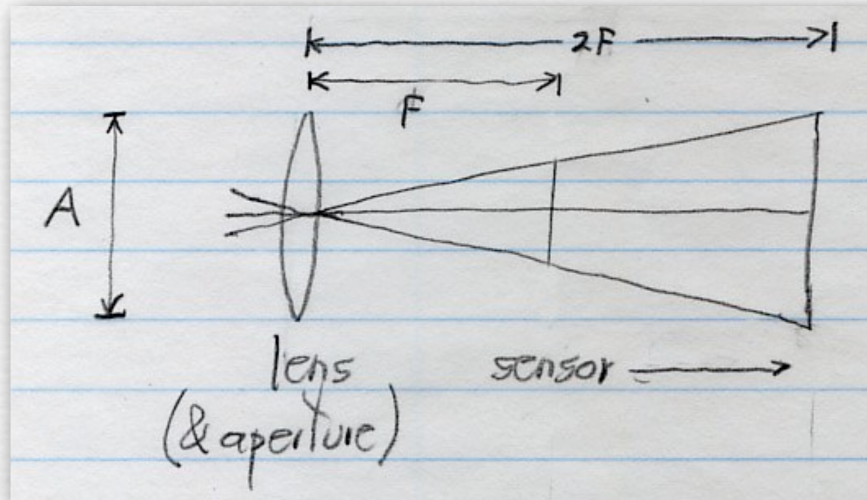
1/2000 sec

Aperture

- irradiance on sensor is proportional to
  - square of aperture diameter A
  - inverse square of distance to sensor (~ focal length f)

Irradiance on sensor (contents of whiteboard)





- As the diameter A of the aperture doubles, its area (hence the light that can get through it) increases by 4× (first drawing).
- Think of the lens as a collection of pinholes, each having a fixed angular field of view (cone in 2nd drawing) determined by the lens design.
- A certain amount of light gets through each pinhole. By conservation of energy, that light will fall on whatever sensor is placed in its path.
- If the distance to the sensor is doubled, the area intersecting the cone increases by 4×, so the light falling per unit area decreases by 4×.

Aperture

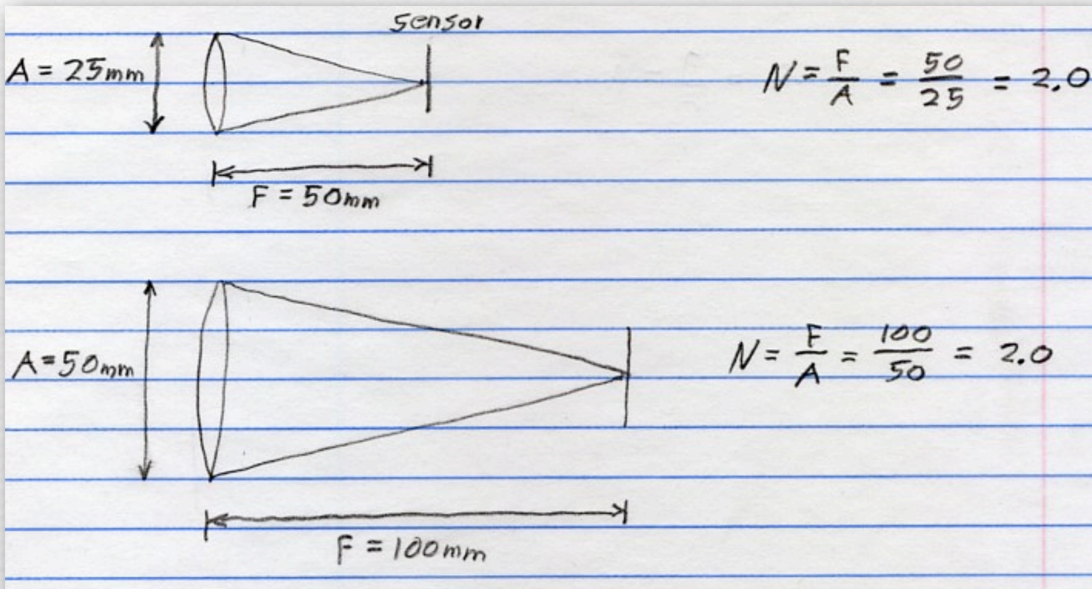
- irradiance on sensor is proportional to
  - square of aperture diameter A
  - inverse square of distance to sensor (~ focal length f)
- so that aperture values give irradiance regardless of lens, *aperture number* N is defined relative to focal length

$$N = \frac{f}{A}$$

  - f/2.0 on a 50mm lens means the aperture is 25mm
  - f/2.0 on a 100mm lens means the aperture is 50mm

  ∴ low F-number (N) on long telephotos require fat lenses

Example F-number calculations (contents of whiteboard)



- A relative aperture size (called F-number or just N) of 2 is sometimes written f/2, reflecting the fact that it is computed by dividing focal length (f) by the absolute aperture diameter (A).
- As this drawing shows, doubling both the absolute aperture diameter (A) and the focal length (f) cancel; leaving the same relative aperture size (N). In this example, both lenses are f/2.

Aperture

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  - square of aperture diameter A
  - inverse square of distance to sensor (~ focal length f)
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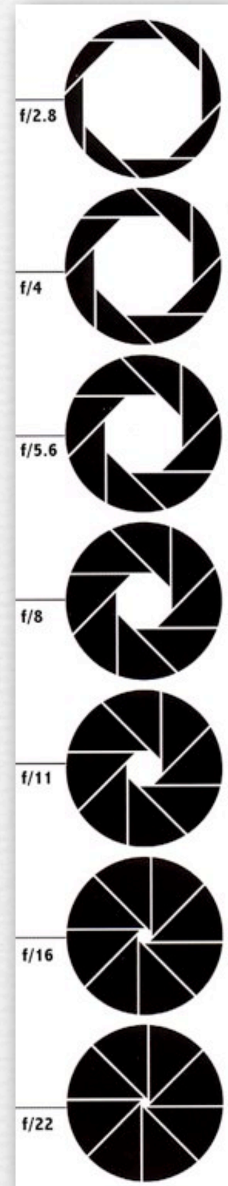
  ∴ low F-number (N) on long zooms require fat lenses

- doubling N reduces A by 2×, hence light by 4×
  - going from f/2.0 to f/4.0 cuts light by 4×
  - to cut light by 2×, increase N by √2

f/2.8
f/4
f/5.6
f/8
f/11
f/16
f/22



(London)

Main side-effect of aperture

- depth of field
- doubling N (two f/stops) doubles depth of field

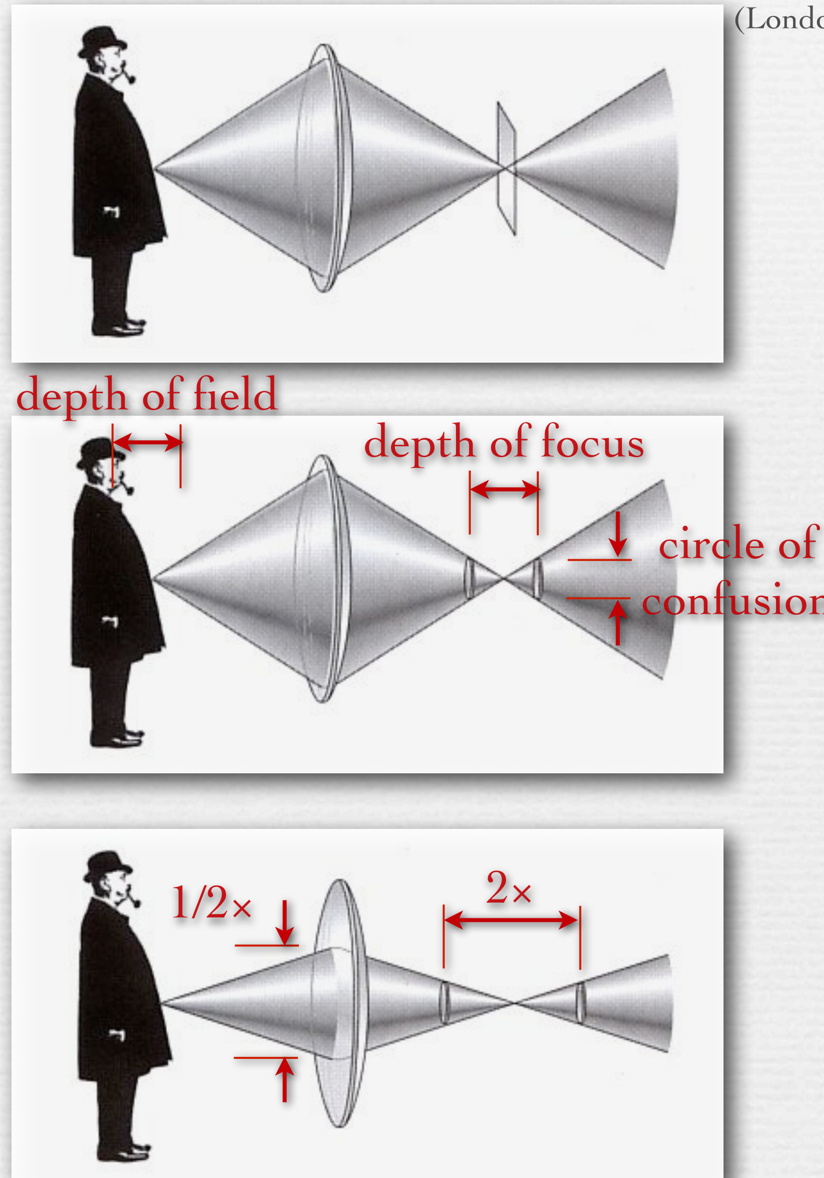
(London)

Depth of field (briefly)

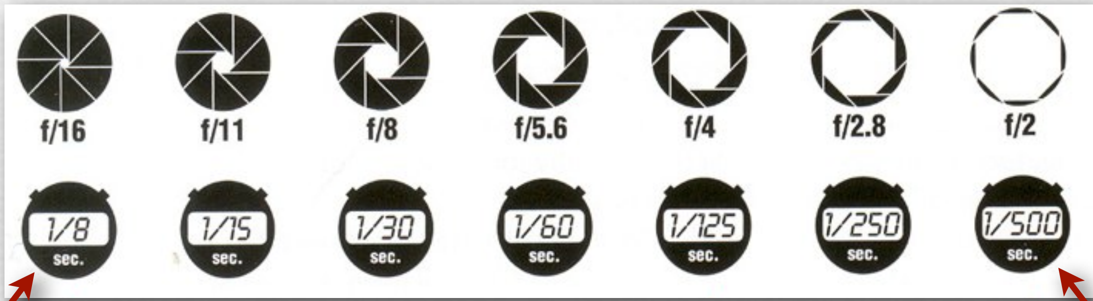
This figure isn't quite right; we'll fix it next week

- a point in the scene is focused at a point on the sensor
- if we move the sensor in depth, the point becomes blurred
- if it blurs too much, it exceeds our allowable *circle of confusion*
- the depth where this happens is called the *depth of focus*
- this corresponds in the scene to a *depth of field*
- halving the aperture diameter doubles the depth of field

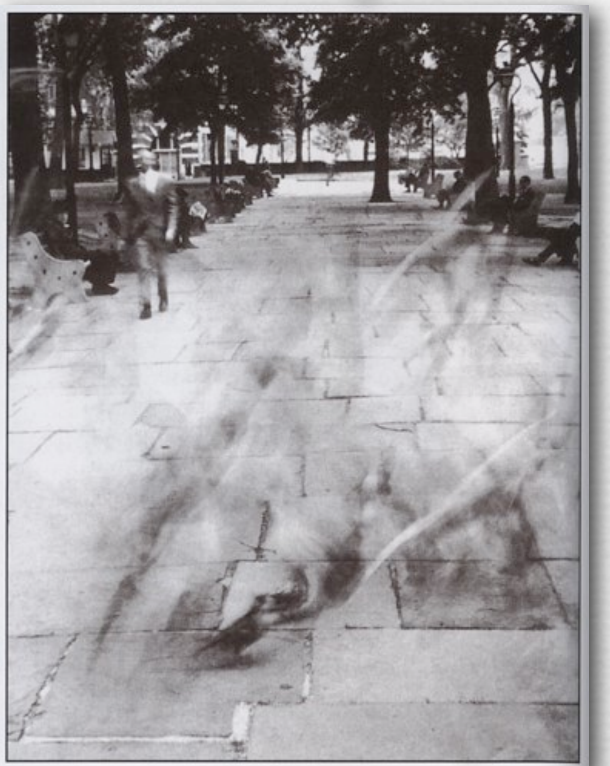
(London)

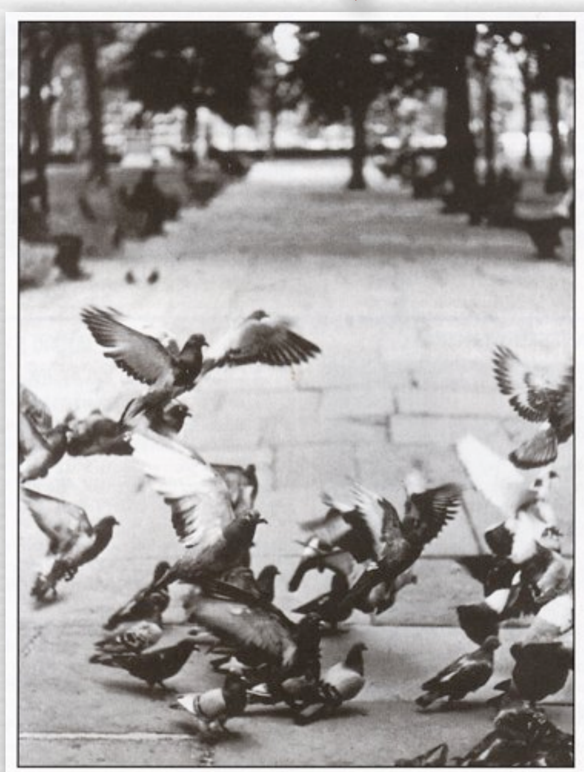


Trading off motion blur and depth of field



(London)



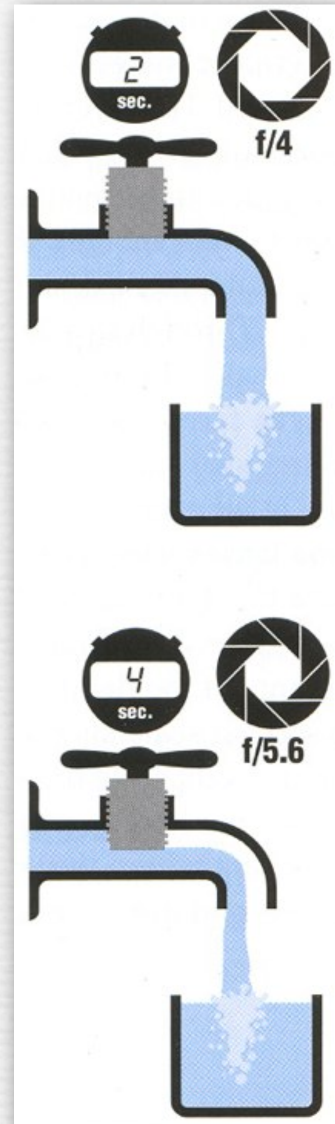


Recap

To save time I skipped this recap slide in class.

- ✦ H = E × T
- ✦ exposure = irradiance × time
- ✦ irradiance (E)
  - controlled by the aperture
  - lowering by one f/stop doubles H
  - lowering by two f/stops doubles depth of field
- ✦ exposure time (T)
  - controlled by the shutter
  - doubling exposure time doubles H
  - doubling exposure time doubles motion blur

(London)

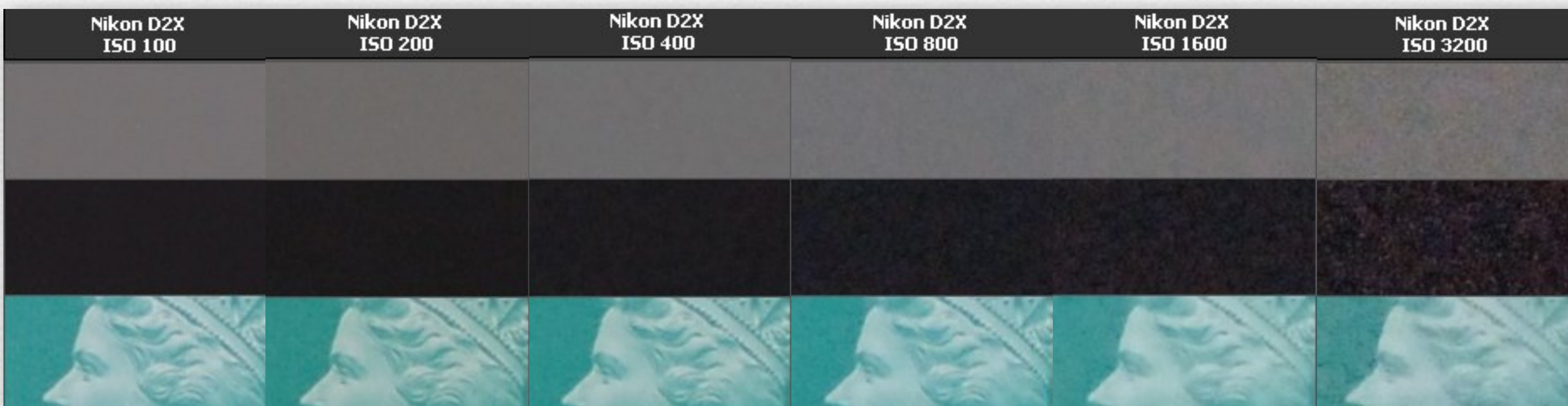


Questions?

Sensitivity (ISO)

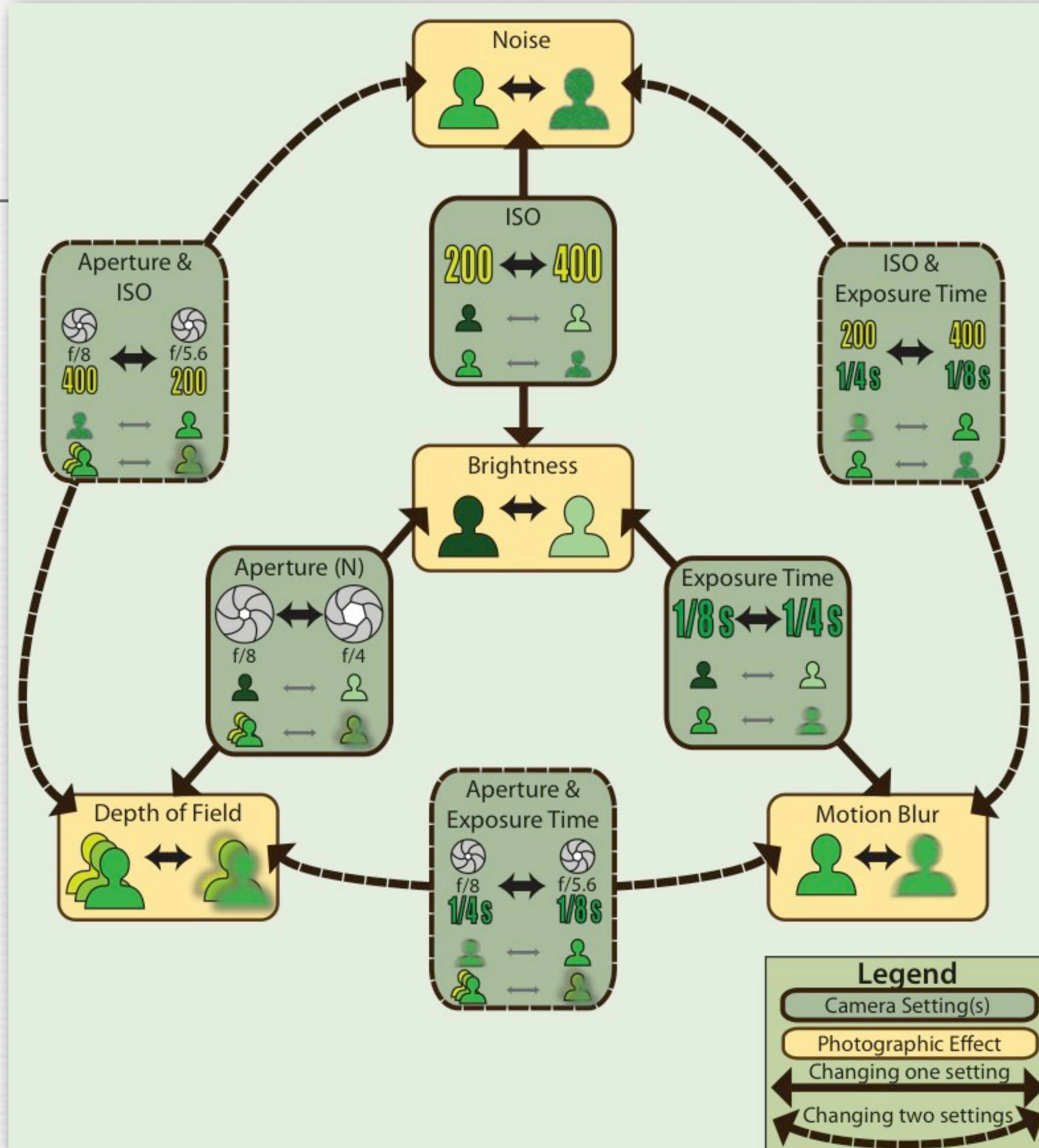
- third variable for exposure
- film: trade sensitivity for grain
- digital: trade sensitivity for noise
  - multiply signal before analog-to-digital conversion
  - linear effect (200 ISO needs half the light as 100 ISO)

more in noise lecture



(dpreview)

Tradeoffs affecting brightness



The Av and Tv buttons on the app, which mean Aperture priority and Shutter priority, are explained in the narrative that accompanies the Flash applet. I encourage you to play with the applet and read the narrative. The URL is below.

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/exposure.html

(Eddy Talvala)

Slide credits

- Steve Marschner
- Fredo Durand
- Eddy Talvala

- Cole, A., *Perspective*, Dorling Kindersley, 1992.
- Kemp, M.,*The Science of Art*,Yale University Press, 1990.
- Hecht, E., *Optics* (4th ed.), Pearson / Addison-Wesley, 2002.
- Renner, E., *Pinhole Photography* (2nd ed.), Focal Press, 2000.
- London, Stone, and Upton, *Photography* (9th ed.), Prentice Hall, 2008.
- D'Amelio, J., *Perspective Drawing Handbook*, Tudor Press, 1964.
- Dubery, F., Willats, J., *Perspective and other drawing systems*, Van Nostrand Reinhold, 1972.
- Kingslake, R. *Optics in Photography*, SPIE Press, 1992.
- http://dpreview.com