Image formation

CS 178, Spring 2010

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Outline

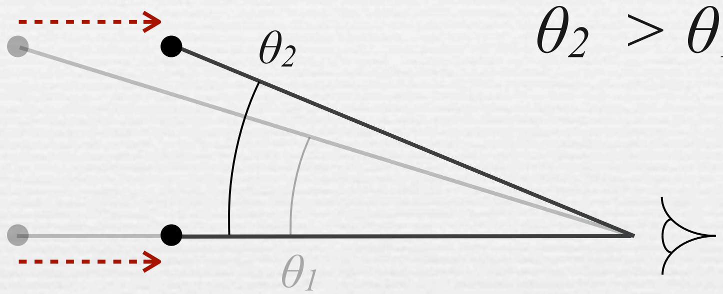
- perspective
  - natural versus linear perspective
  - vanishing points
- image formation
  - pinhole cameras
  - lenses
- exposure
  - shutter speed
  - aperture
  - ISO
- choosing a camera

The laws of perspective

- common assumptions
  1. Light leaving an object travels in straight lines.
  2. These lines converge to a point at the eye.

- natural perspective (Euclid, 3rd c. B.C.)
  3a. More distant objects subtend smaller visual angles.

The laws of perspective



$\theta_2 > \theta_1$

I wrote "≥" on the board, but if the points move towards the viewer by any non-zero amount, θ_2 will definitely become greater than θ_1, so I should have written ">".

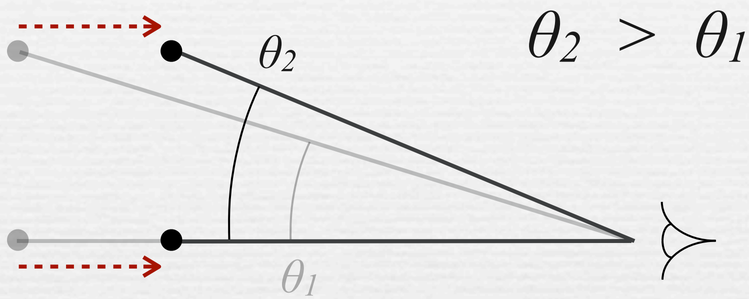
- natural perspective (Euclid, 3rd c. B.C.)
  3a. More distant objects subtend smaller visual angles.

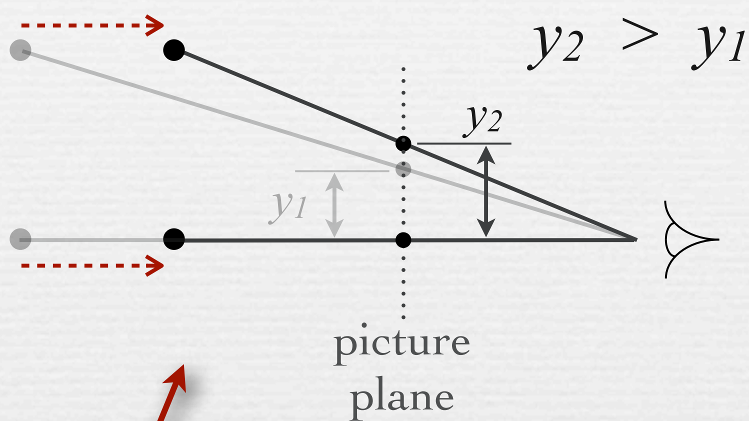
Roman wall paintings

from Villa Publius Fannius Synistor,
Boscoreale, Pompeii (c. 40 B.C.)

Still life with peaches, from
Herculaneum (before 79 A.D.)

The laws of perspective





- natural perspective (Euclid, 3rd c. B.C.)

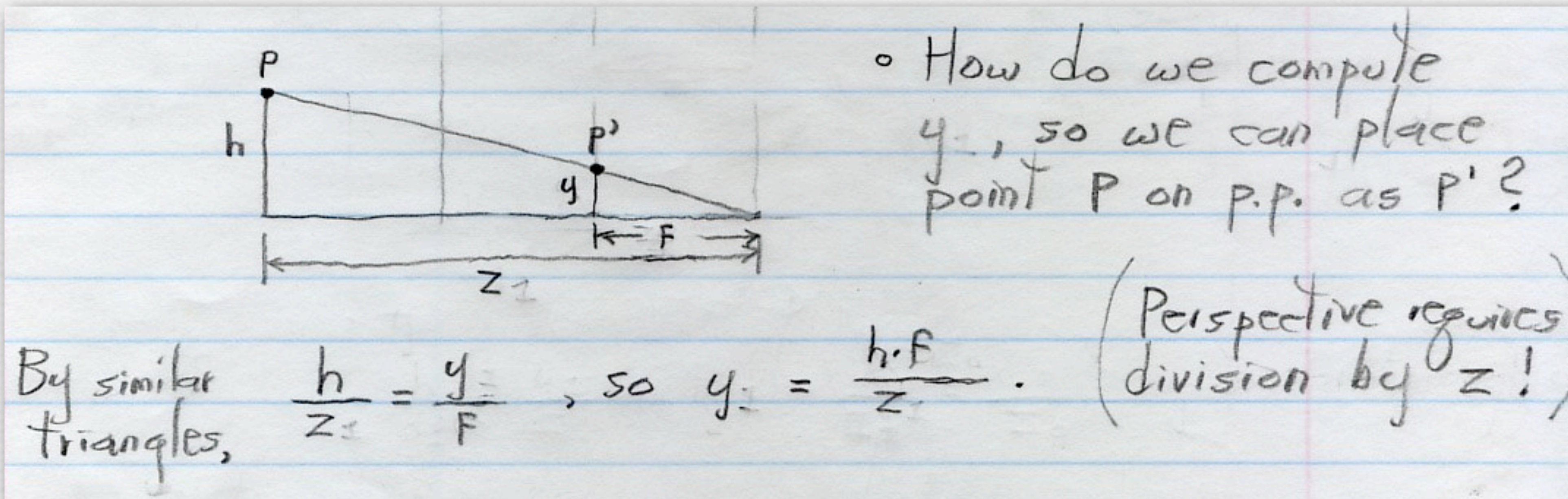
  3a. More distant objects subtend smaller visual angles.

- linear perspective (Filippo Brunelleschi, 1413)

  3b. A perspective image is formed by the intersection of these lines with a "picture plane" (the canvas).

Projection onto picture plane (contents of whiteboard)

To make the online slide set as complete as possible, I 'll occasionally include sketches from my private notes, showing derivations or drawings that I've made in class on the whiteboard. I hope you can pardon the informality of these sketches.

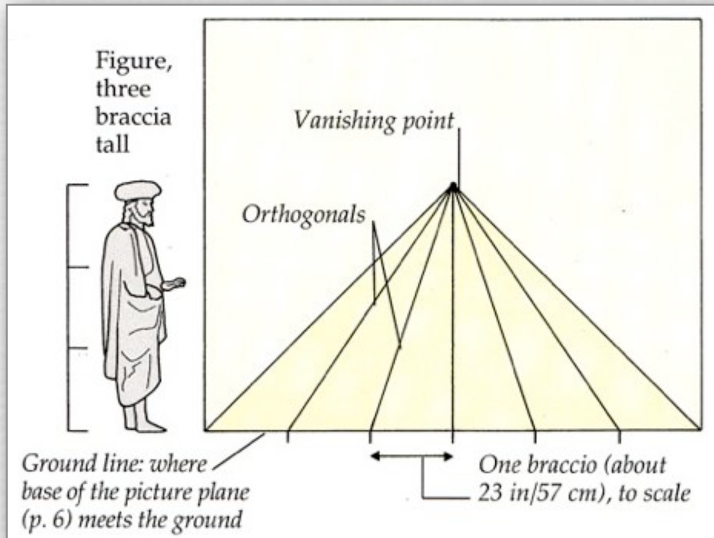


Filippo Brunelleschi, dome of the cathedral, Florence (1419)

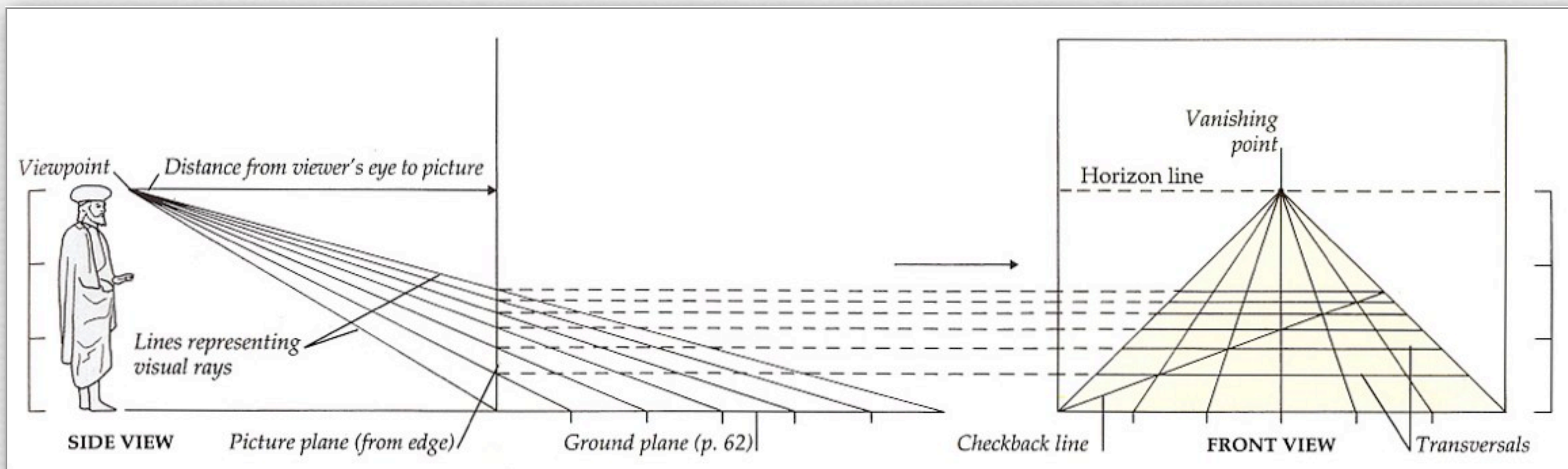
The problem of drawing pavimento

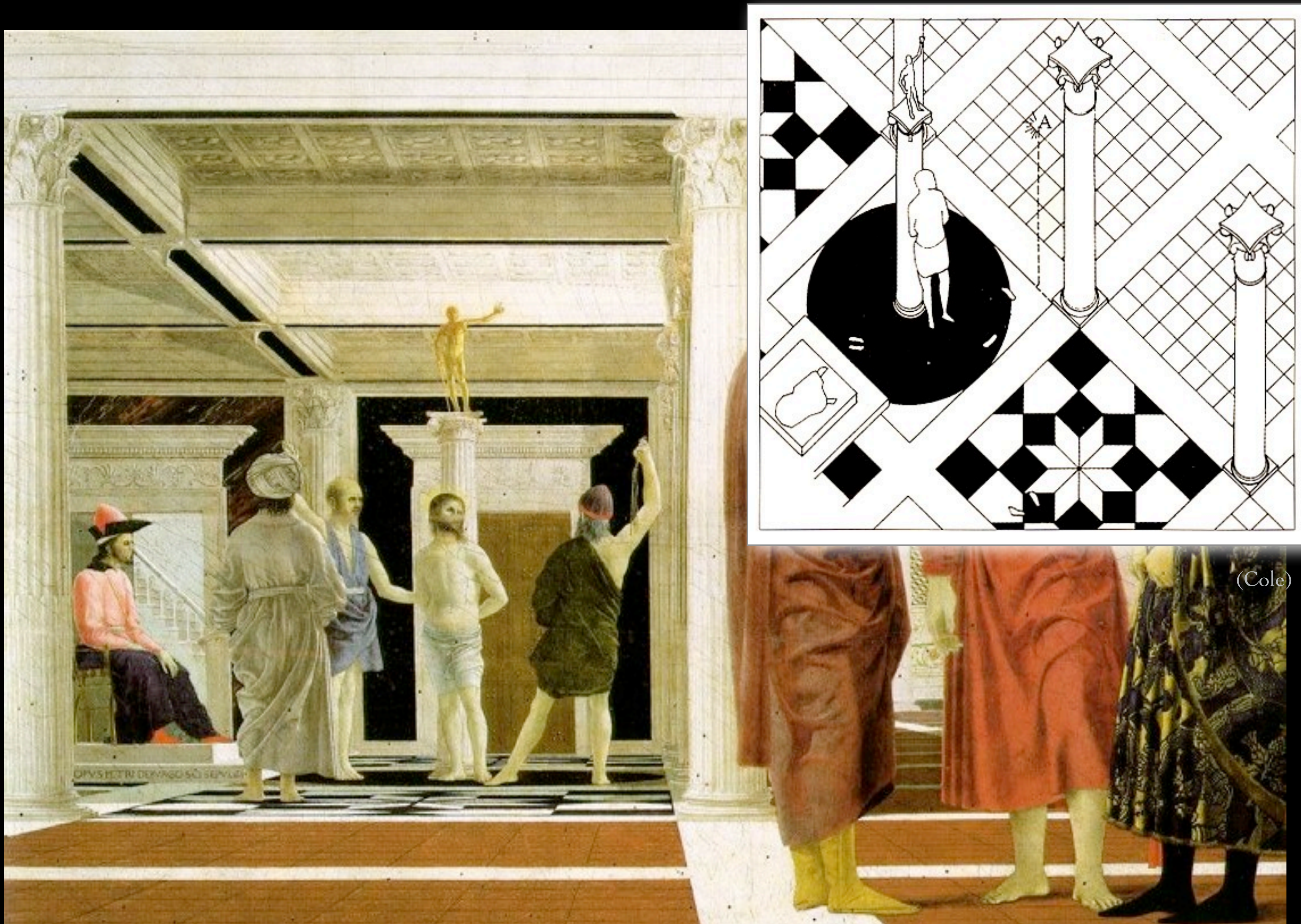
Giovanni de Paolo, Birth of St. John the Baptist (1420)

Alberti's method (1435)



(Cole)



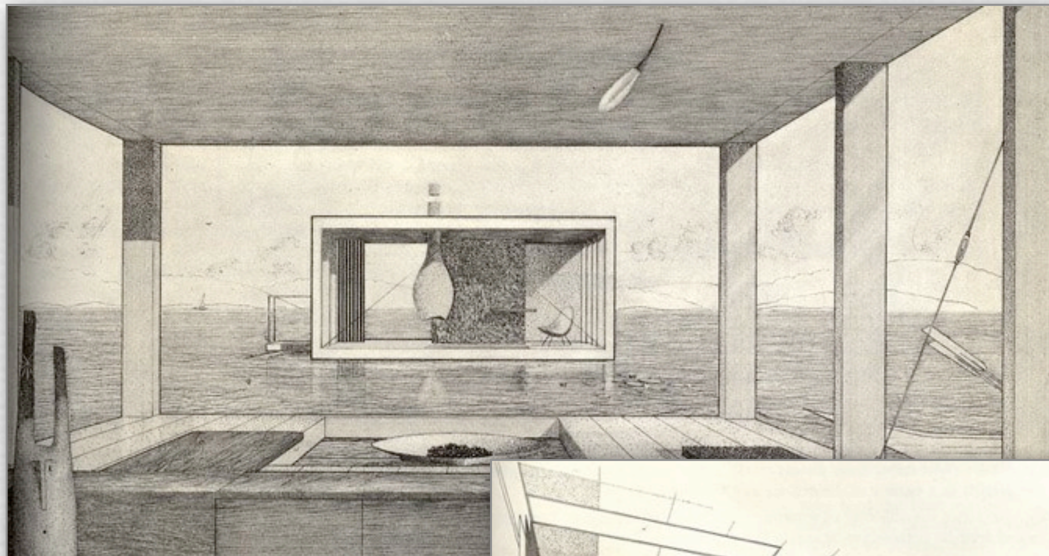


(Cole)

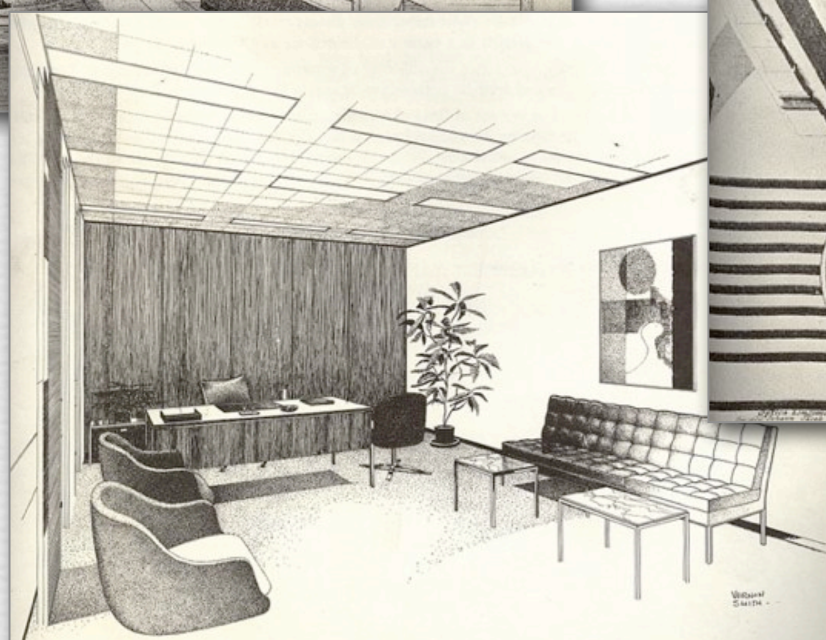
Piero della Francesca, The Flagellation (c.1460)

Vanishing points

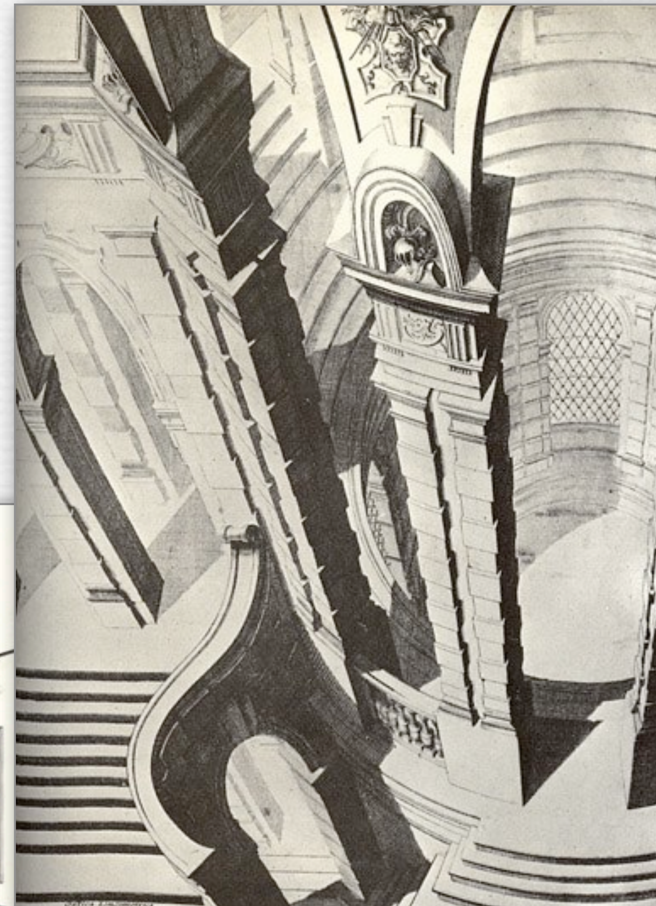
Q. How many vanishing points can there be in a perspective drawing?



1-point



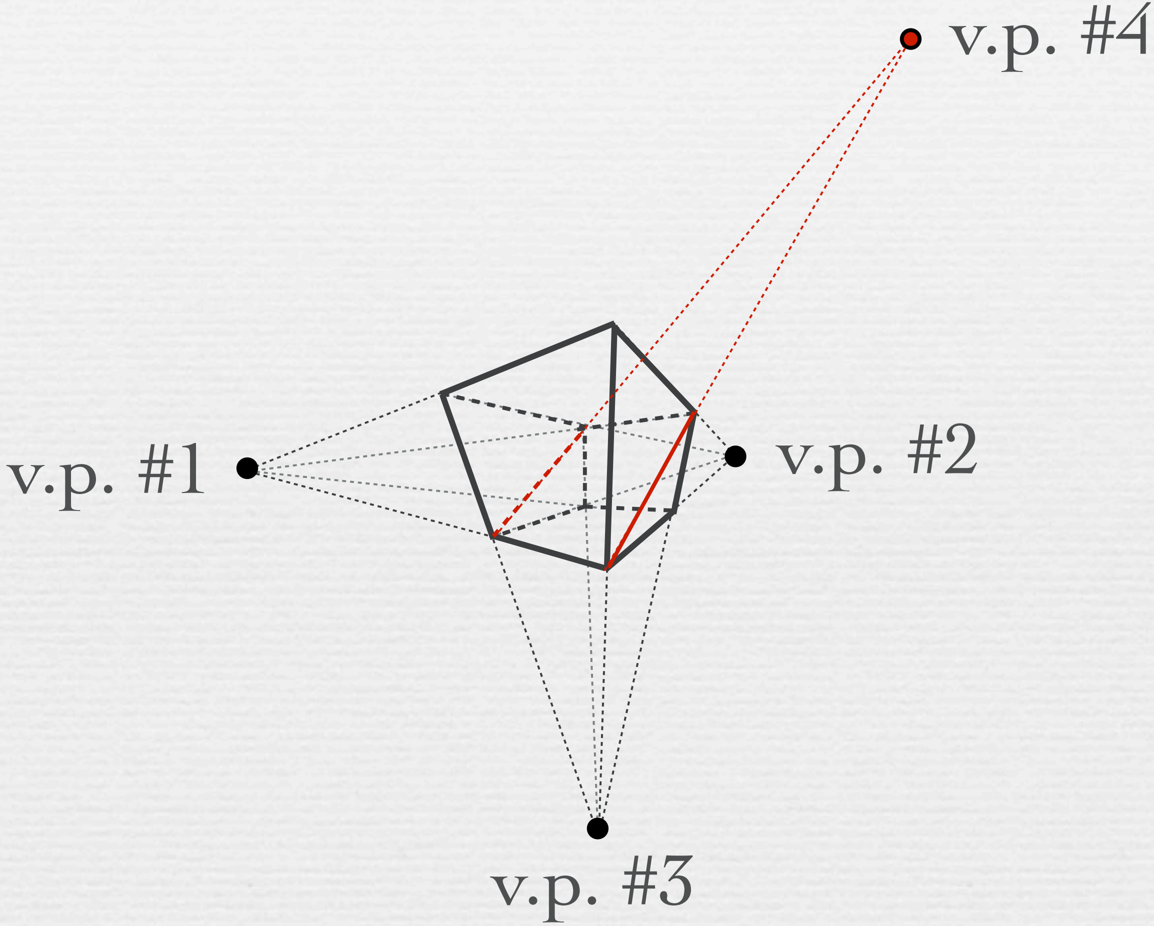
2-point



3-point

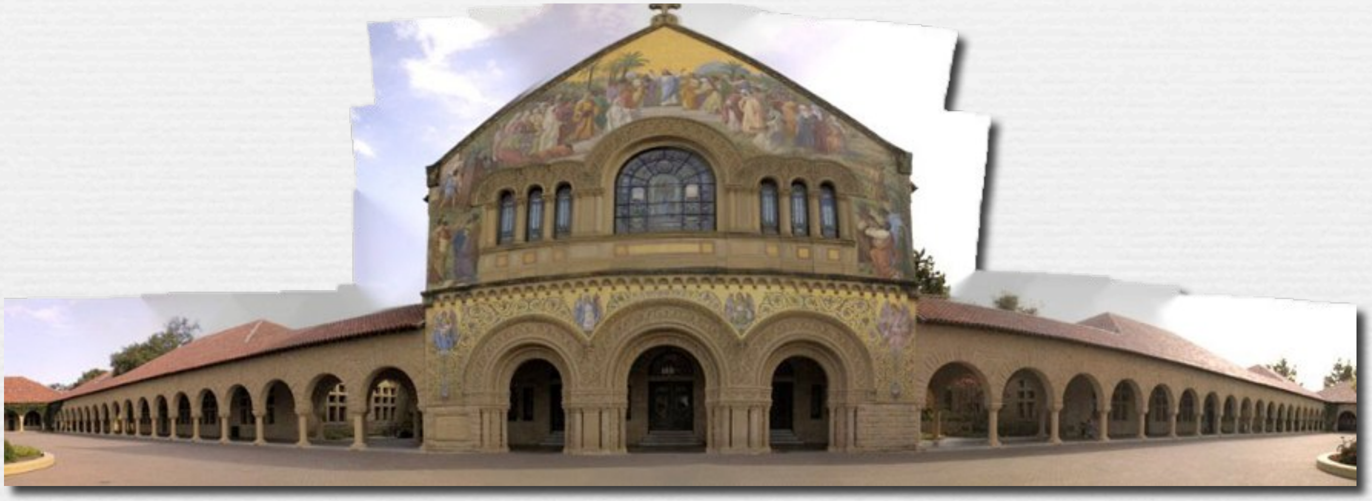
(D'Amelio)

Example of a 4th vanishing point



- each direction of parallel lines will converge to a unique vanishing point

Q. Should the distant ends of a long facade be drawn smaller than its center in a perspective drawing?



?

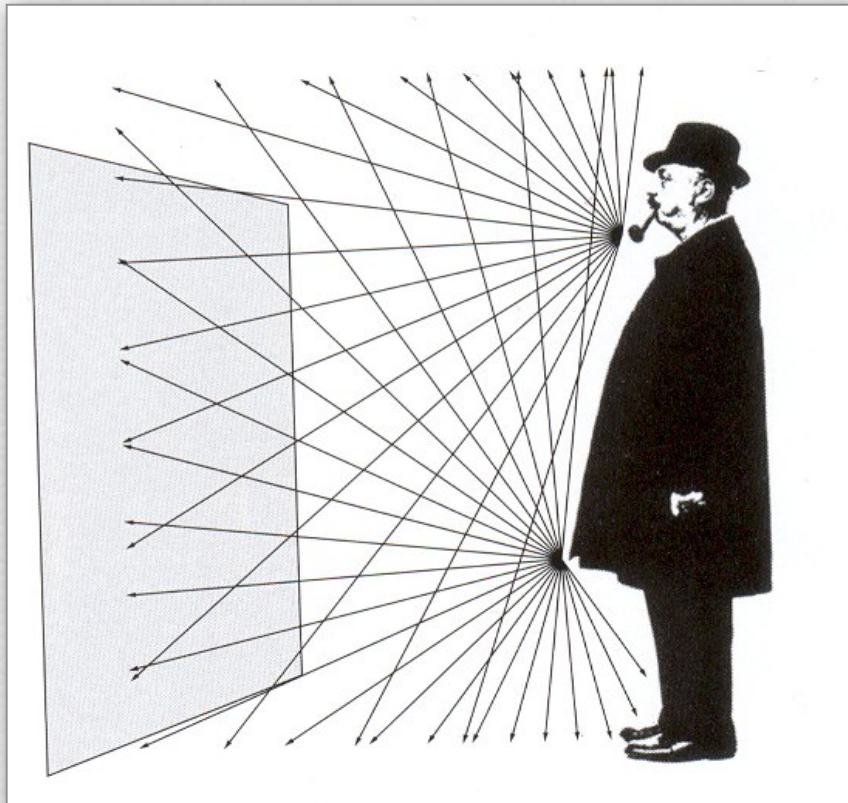
- no, in linear perspectives straight lines remain straight
- lines parallel to the picture plane do not converge
- they appear smaller when you view the drawing, due to natural perspective (angles subtended at eye)

Recap

- natural perspective
  - visual angle subtended by a feature in the world
- linear perspective
  - intersections of lines of sight with a picture plane
  - the correct way to make a drawing on a flat surface
- vanishing points
  - one per direction of line in the scene
  - lines parallel to the picture plane do not converge

Questions?

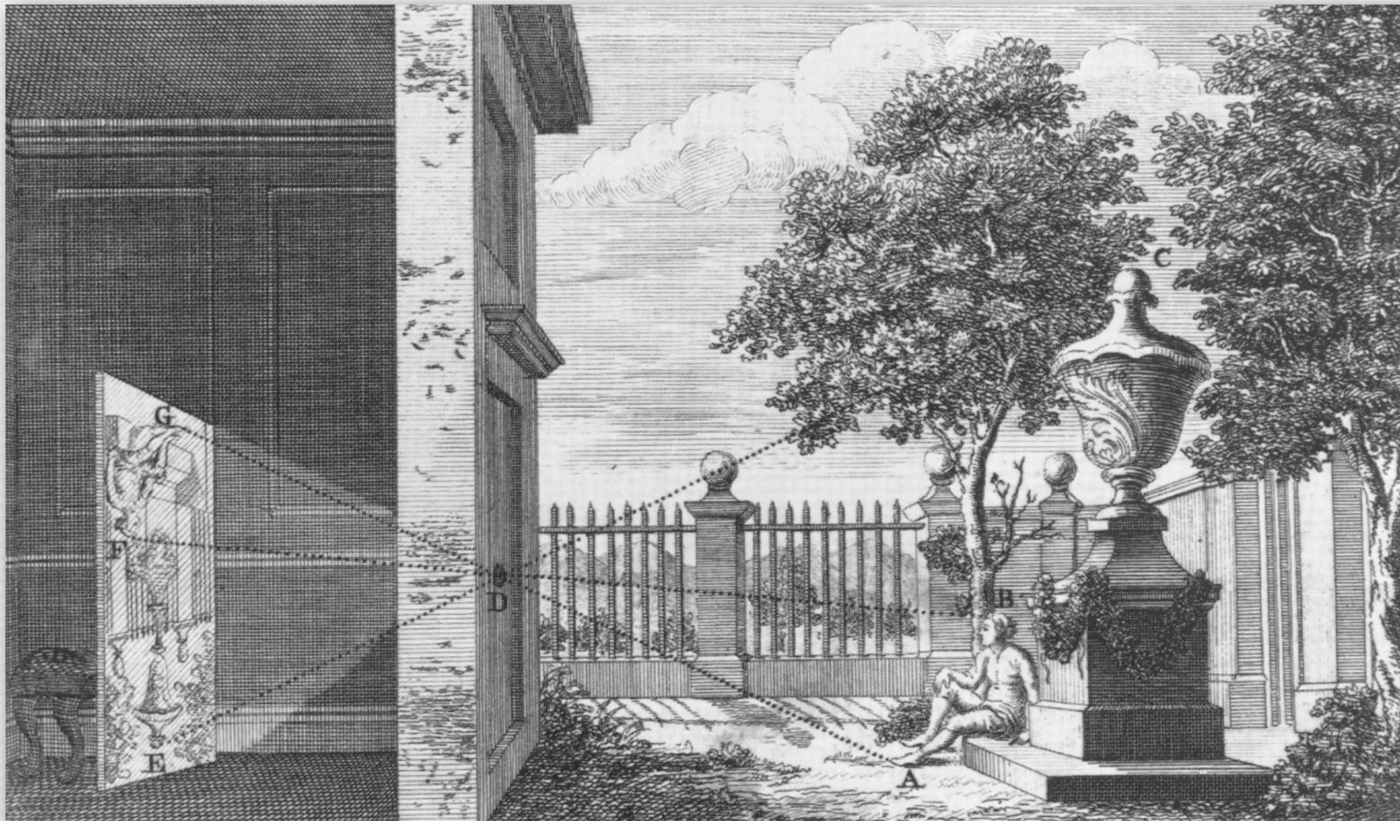
Why not use sensors without optics?



(London)

- each point on sensor would record the integral of light arriving from every point on subject
- all sensor points would record similar colors

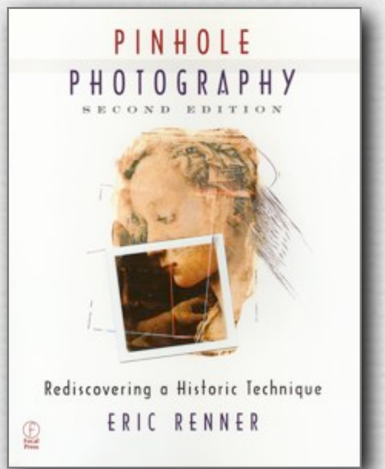
Pinhole camera
(a.k.a. *camera obscura*)



- linear perspective with viewpoint at pinhole
- tilting the picture plane changes the number and location of vanishing points

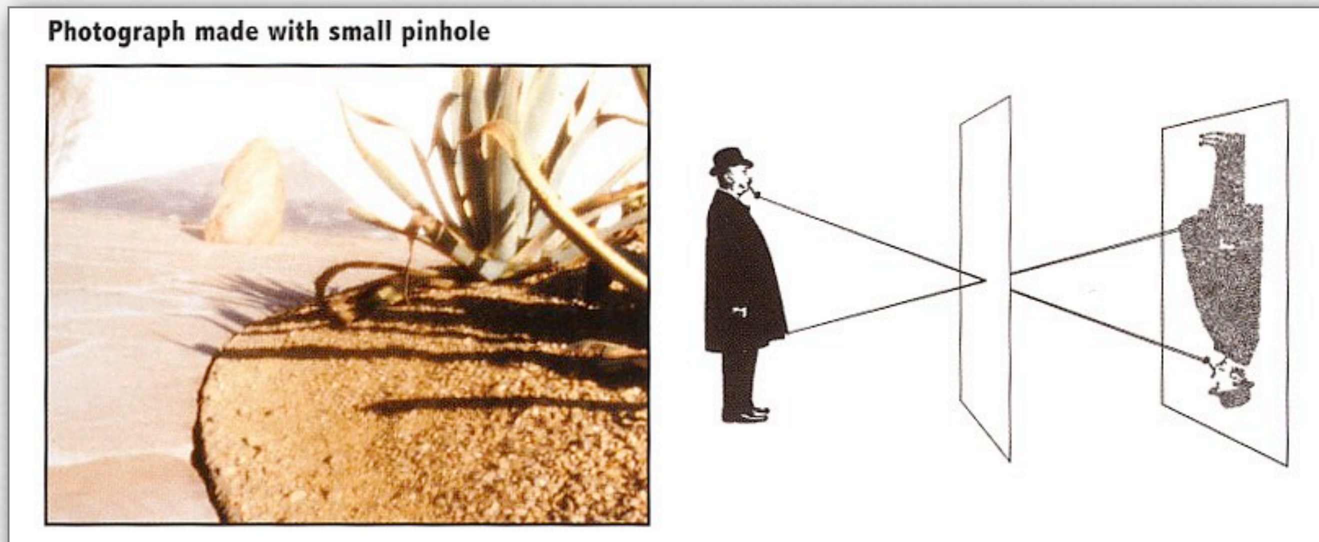
Pinhole photography

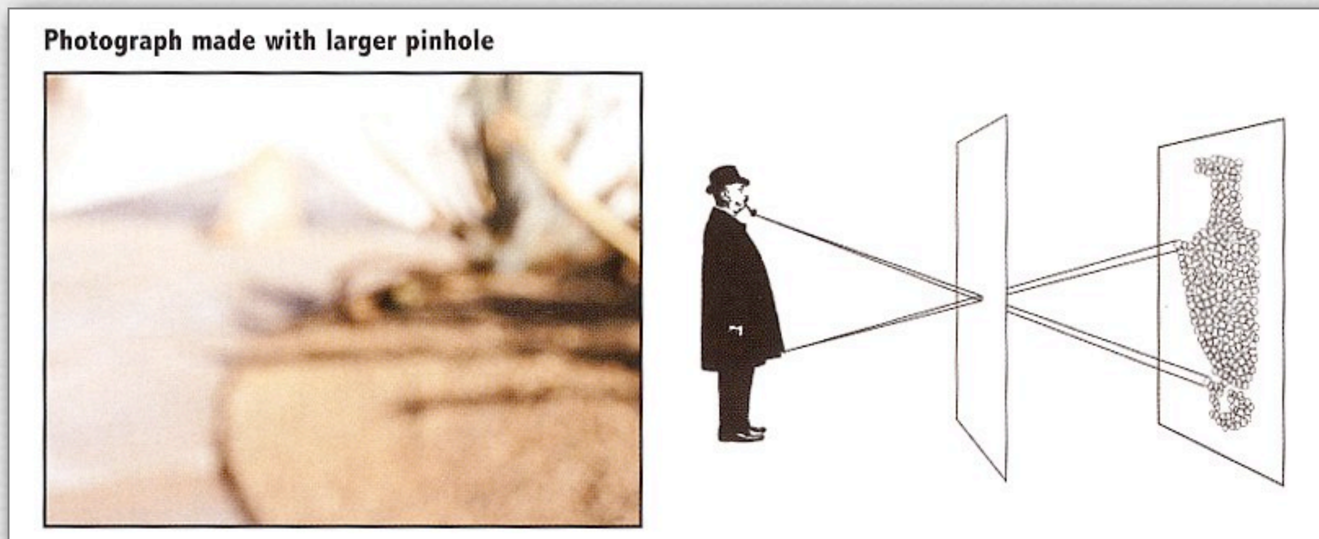
- no distortion
  - straight lines remain straight
- infinite depth of field
  - everything is in focus



(Bami Adedoyin)

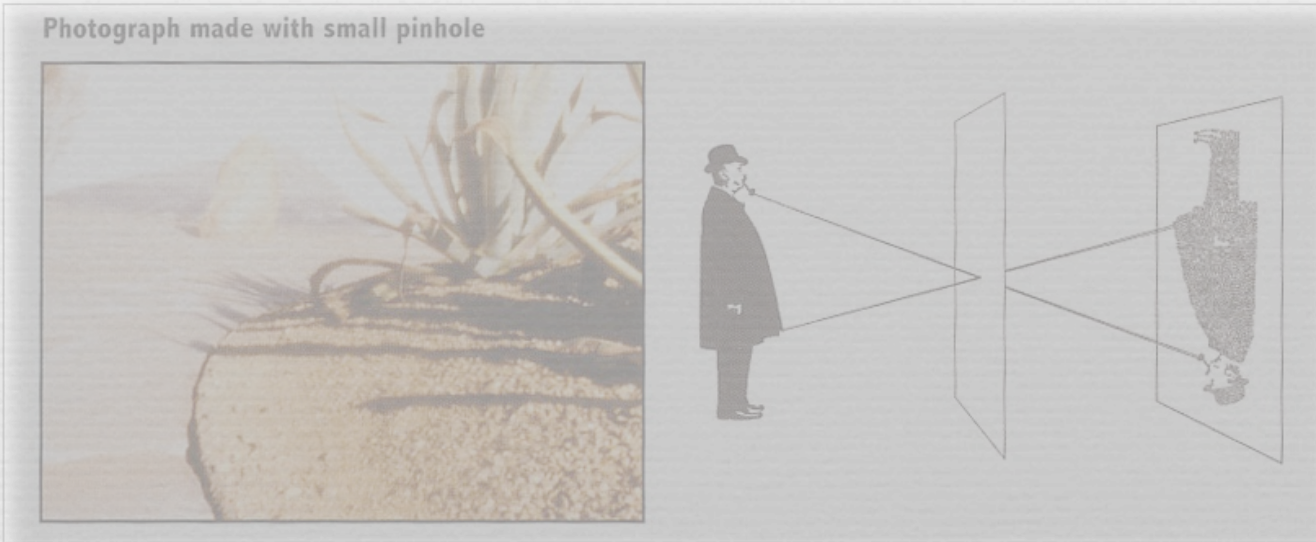
Effect of pinhole size

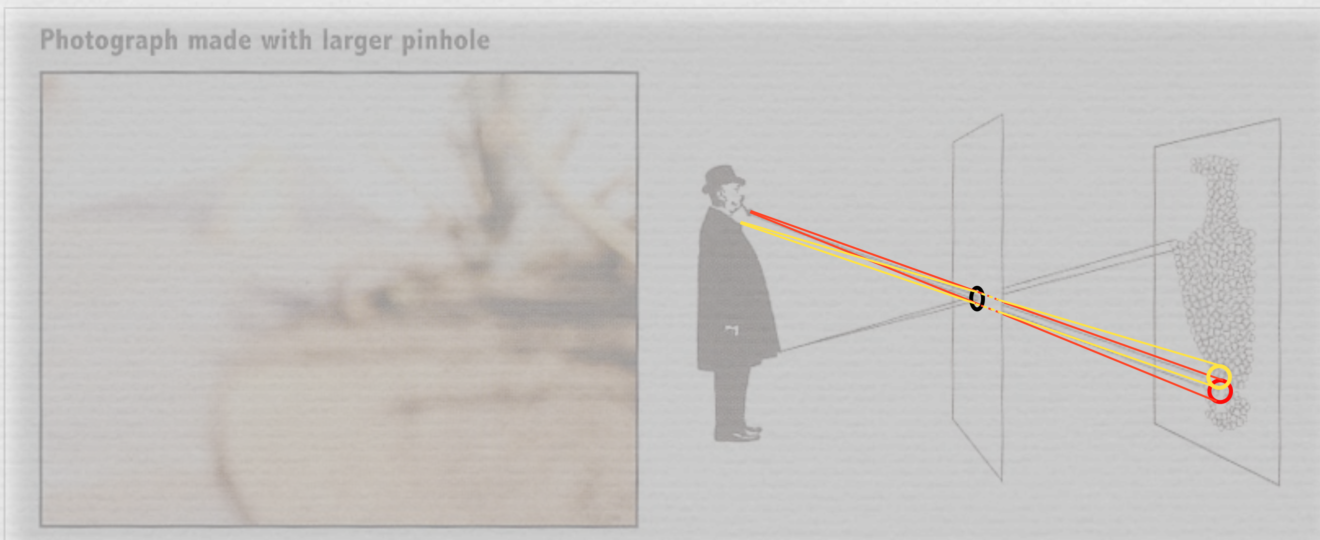




(London)

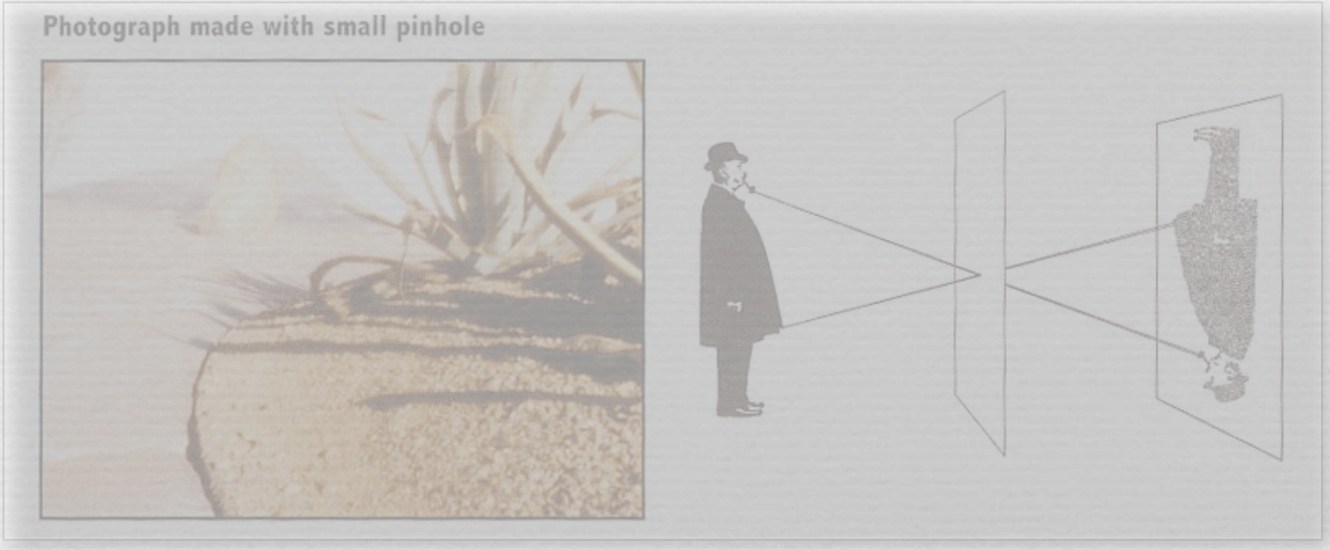
Effect of pinhole size

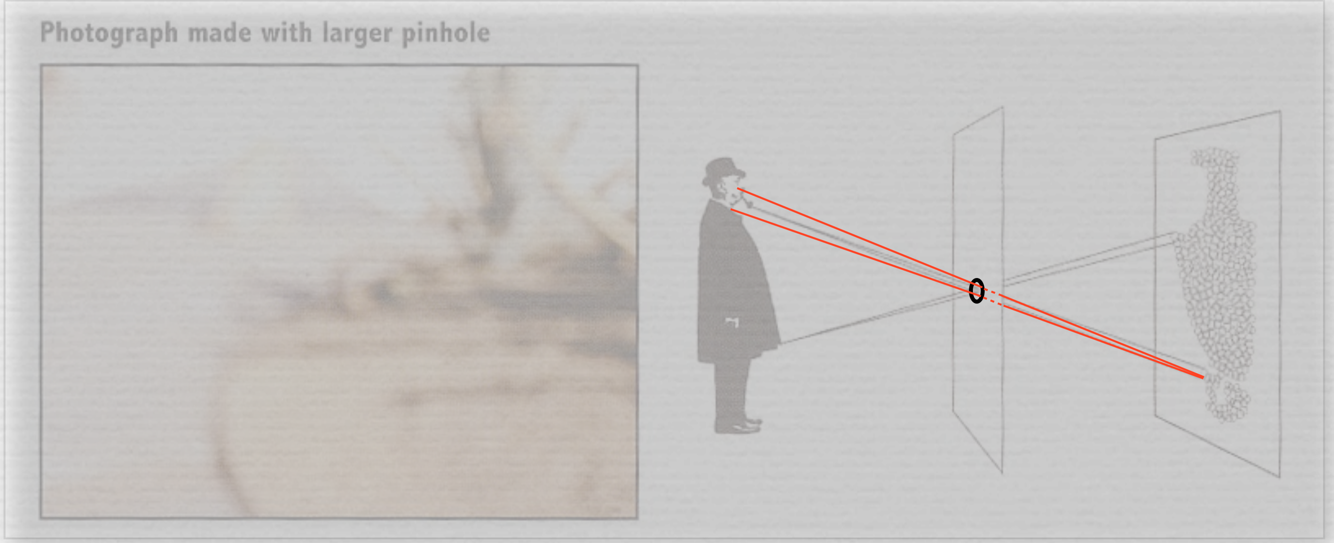




(London)

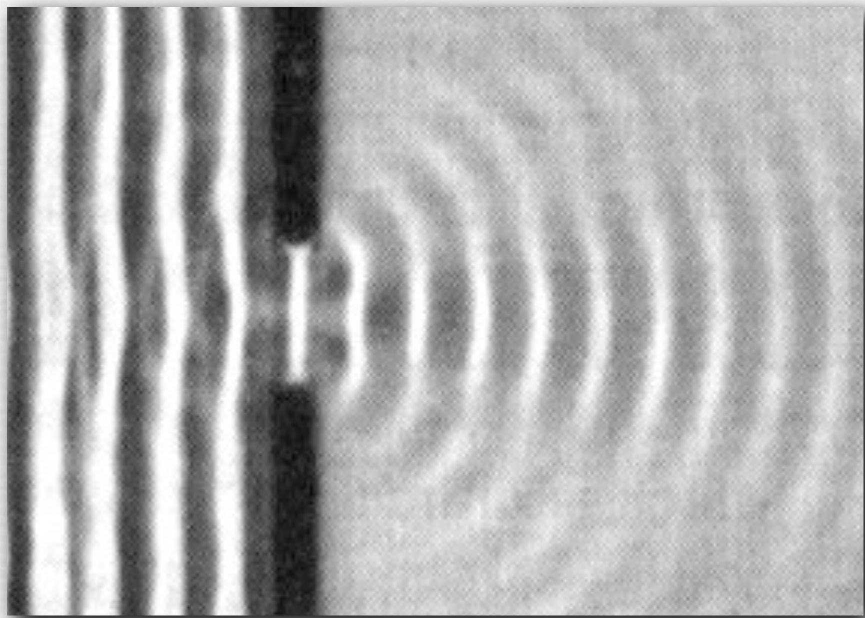
Effect of pinhole size





(London)

Diffraction limit

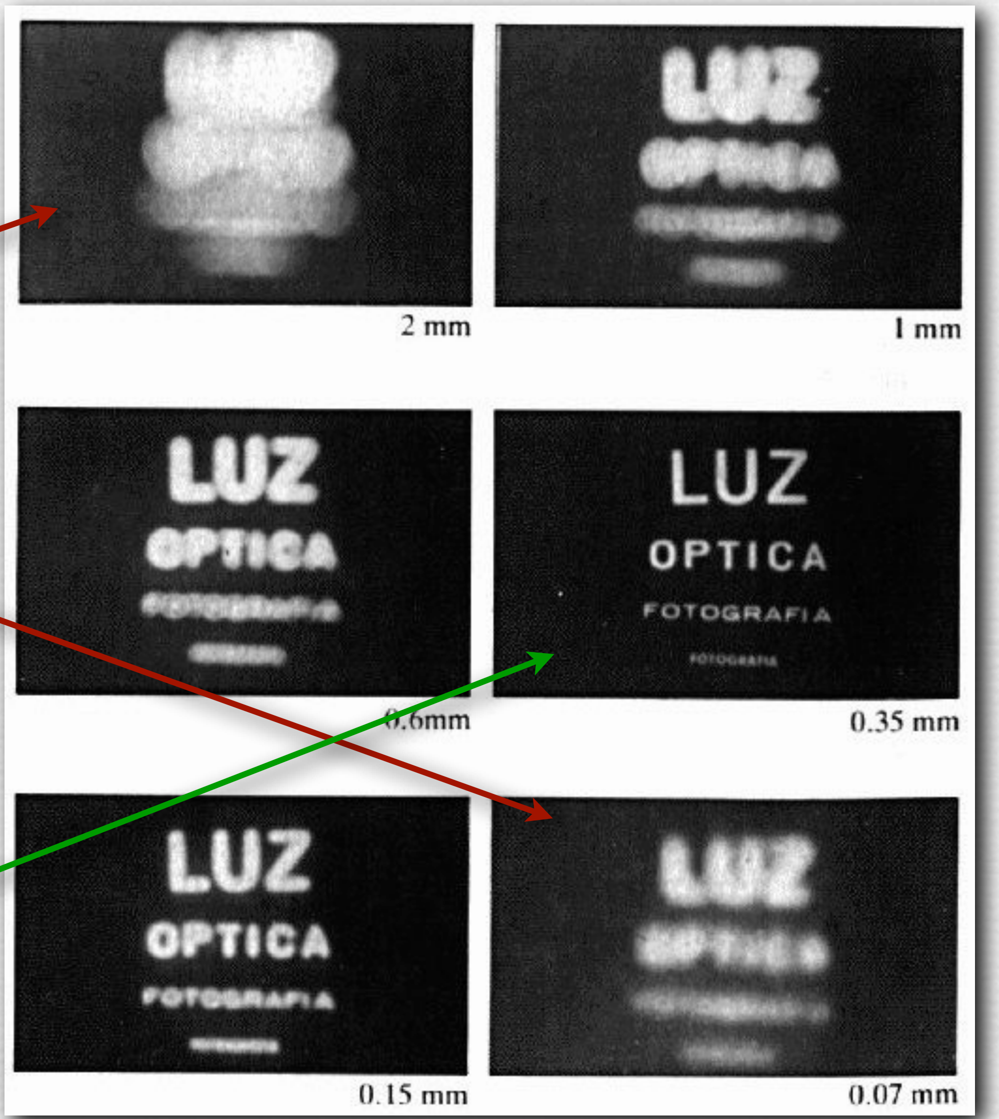


(Hecht)

- due to wave nature of light
- smaller aperture means more diffraction

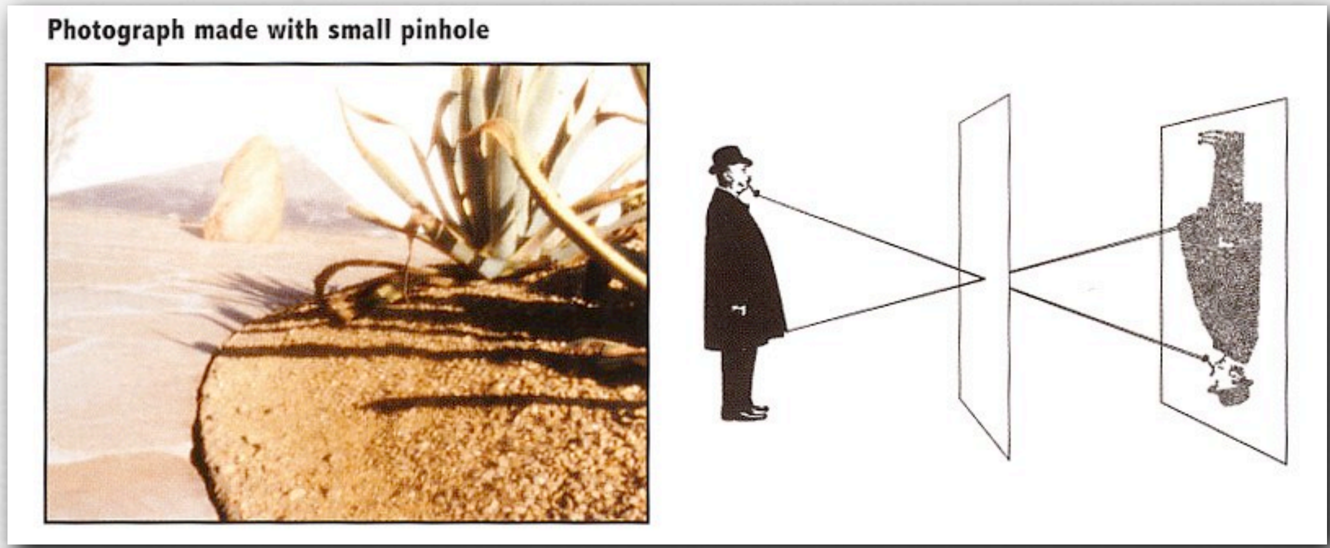
Effect of pinhole size (again)

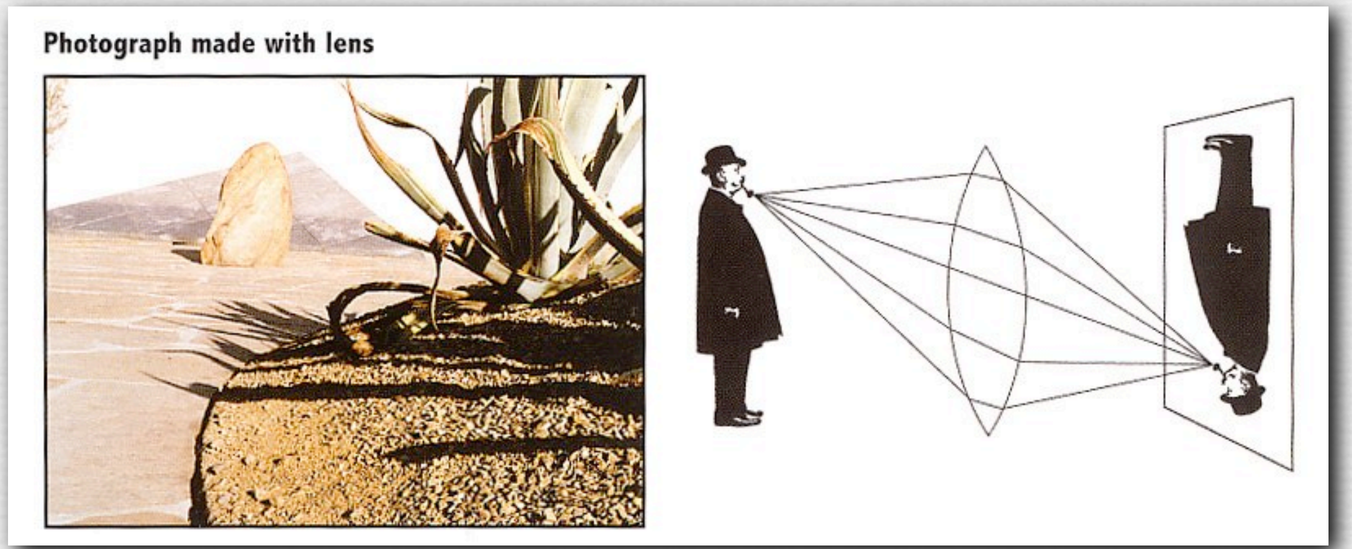
- large pinhole
  gives geometric blur
- small pinhole
  gives diffraction blur
- optimal pinhole
  gives very little light



(Hecht)

Replacing the pinhole with a lens

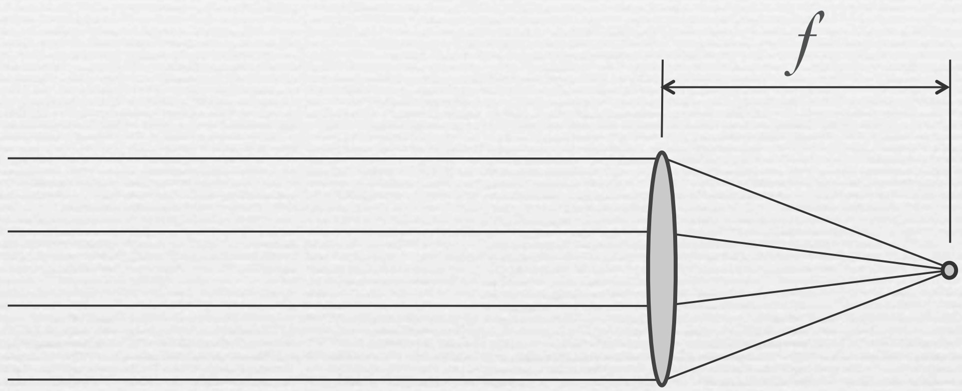




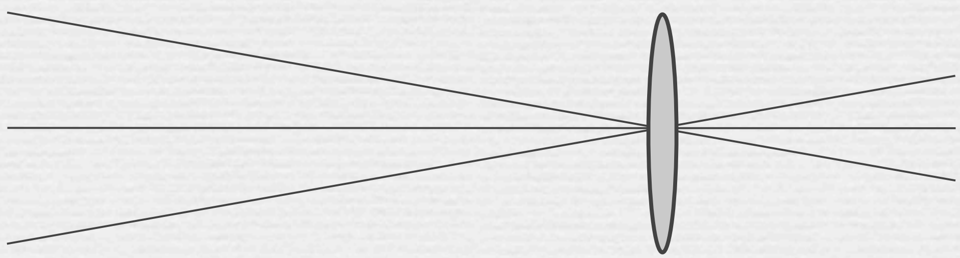
(London)

Geometrical optics

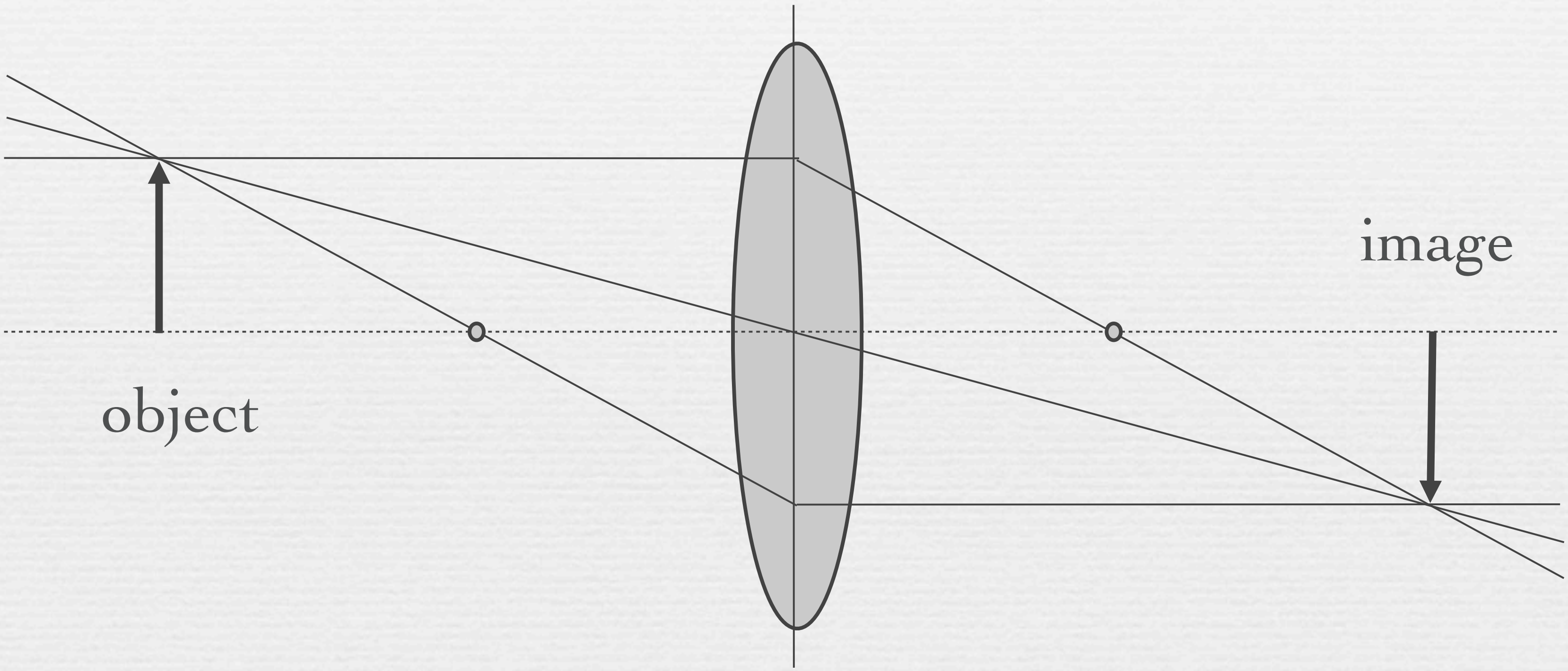
- parallel rays converge to a point located at focal length f from lens



- rays going through center of lens are not deviated
  - hence same perspective as pinhole



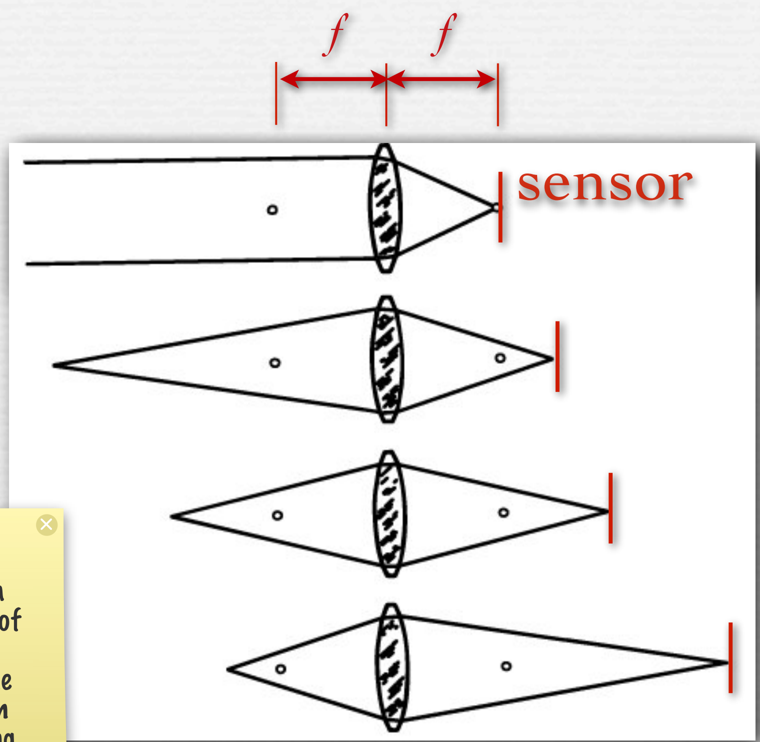
Gauss' ray tracing construction



- rays coming from points on a plane parallel to the lens are focused on another plane parallel to the lens

Changing the focus distance

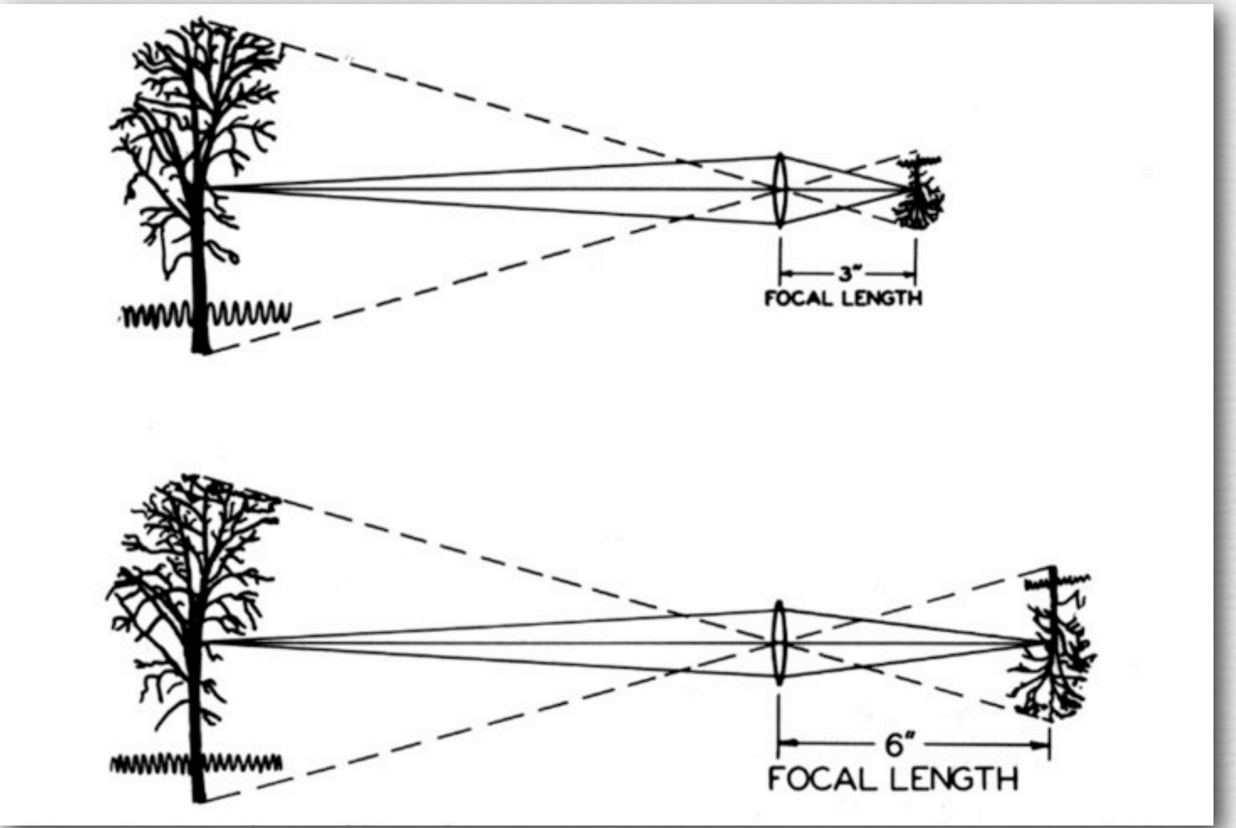
- to focus on objects at different distances, move sensor relative to lens



To clarify a point I made in class, the term "focus distance" usually refers to the distance from the camera (or lens) to a point in the scene, which is on the left side of the lens in this figure. The figure shows that we can modify the focus distance by changing the spacing (inside the camera) between the lens and the sensor, which is on the right side of the lens. This spacing is called something else - sometimes "back focal length" or "back focus distance". When we talk next week about depth of field, we will use the variable "U", and it will mean focus distance, i.e. distance to points in the scene.

Changing the focal length

- weaker lenses have longer focal lengths
- to stay in focus, move the sensor further back



(Kingslake)

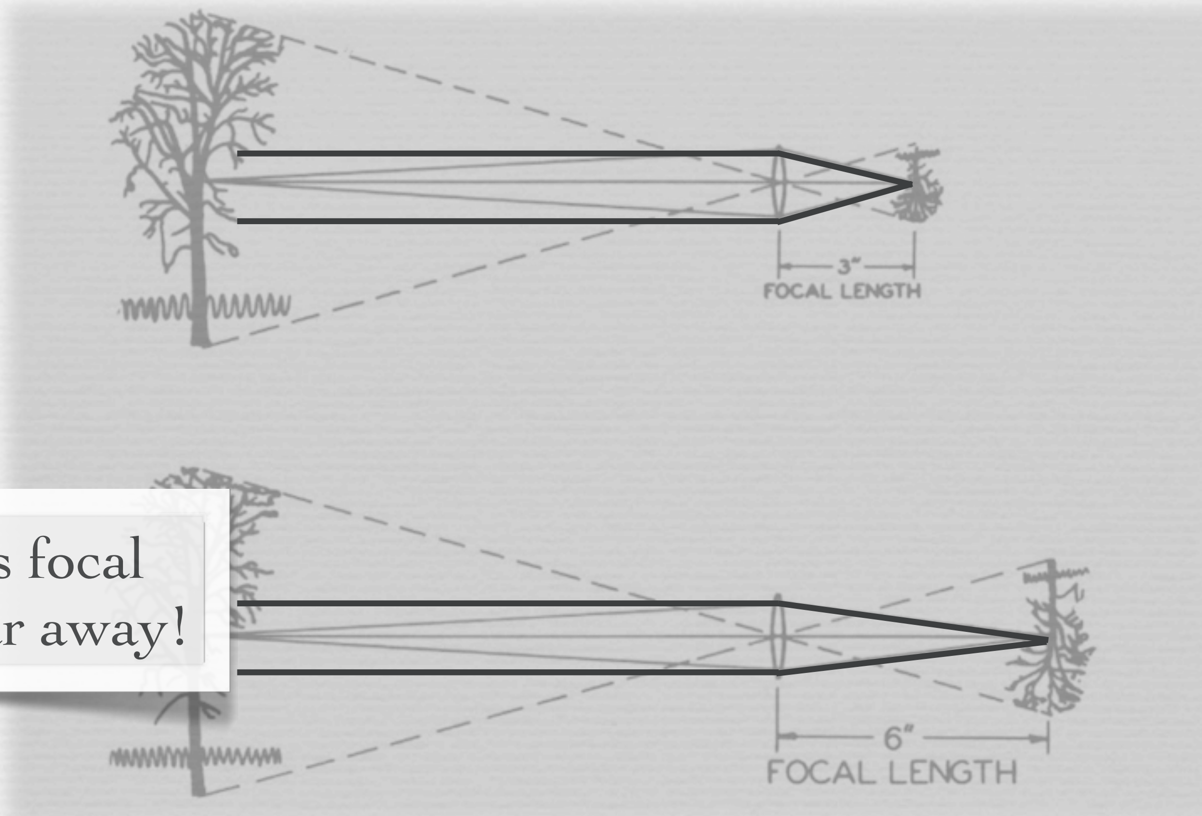
Changing the focal length

This and the following slide added 4/26/10, after someone noticed that Kingslake's famous drawing is actually incorrect!

- weaker lenses have longer focal lengths

- to stay in focus, move the sensor further back

the tree is in focus at the lens focal length only if it's infinitely far away!



(Kingslake)

Changing the focal length

- weaker lenses have longer focal lengths

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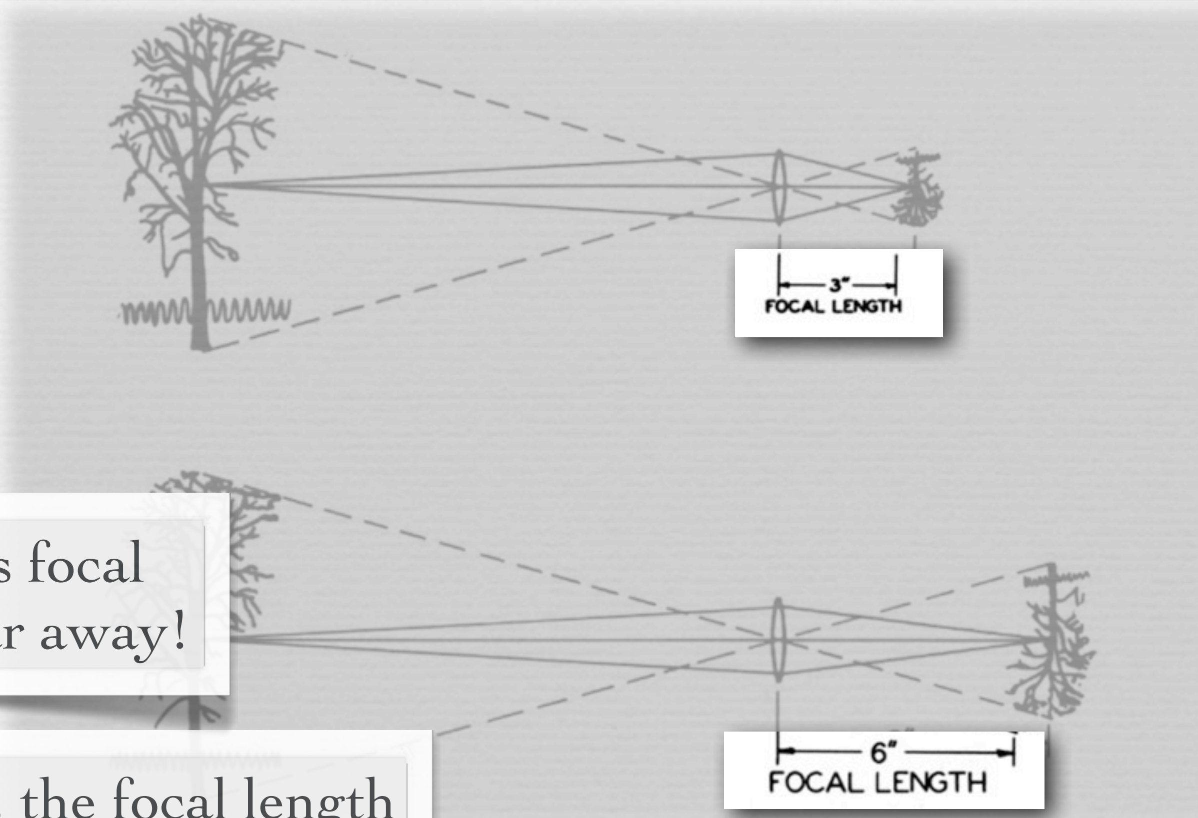


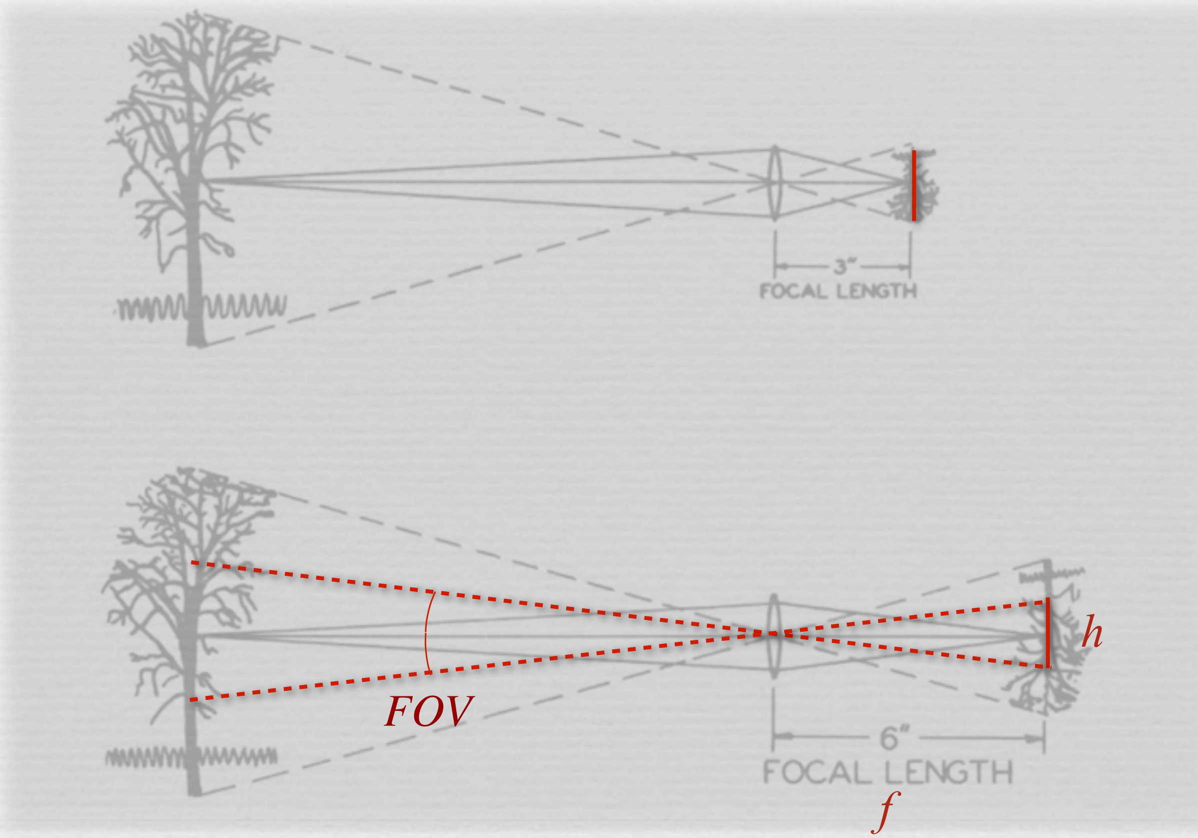
the tree is in focus at the lens focal length only if it's infinitely far away!

for the tree as it's drawn, the focal length should be labeled as slightly smaller than the "image distance", as we'll see later

(Kingslake)

Changing the focal length

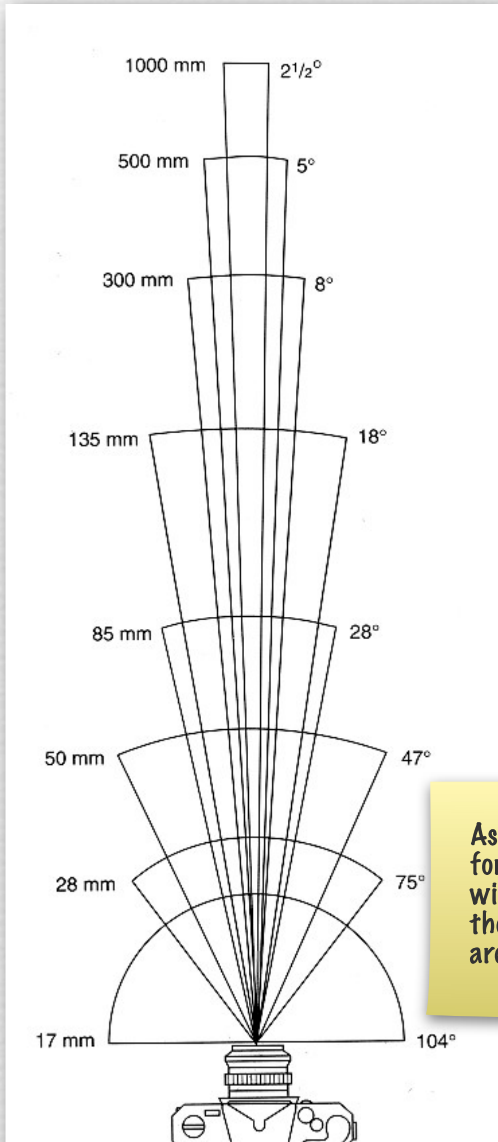
- weaker lenses have longer focal lengths
- to stay in focus, move the sensor further back
- if the sensor size is constant, the field of view becomes smaller

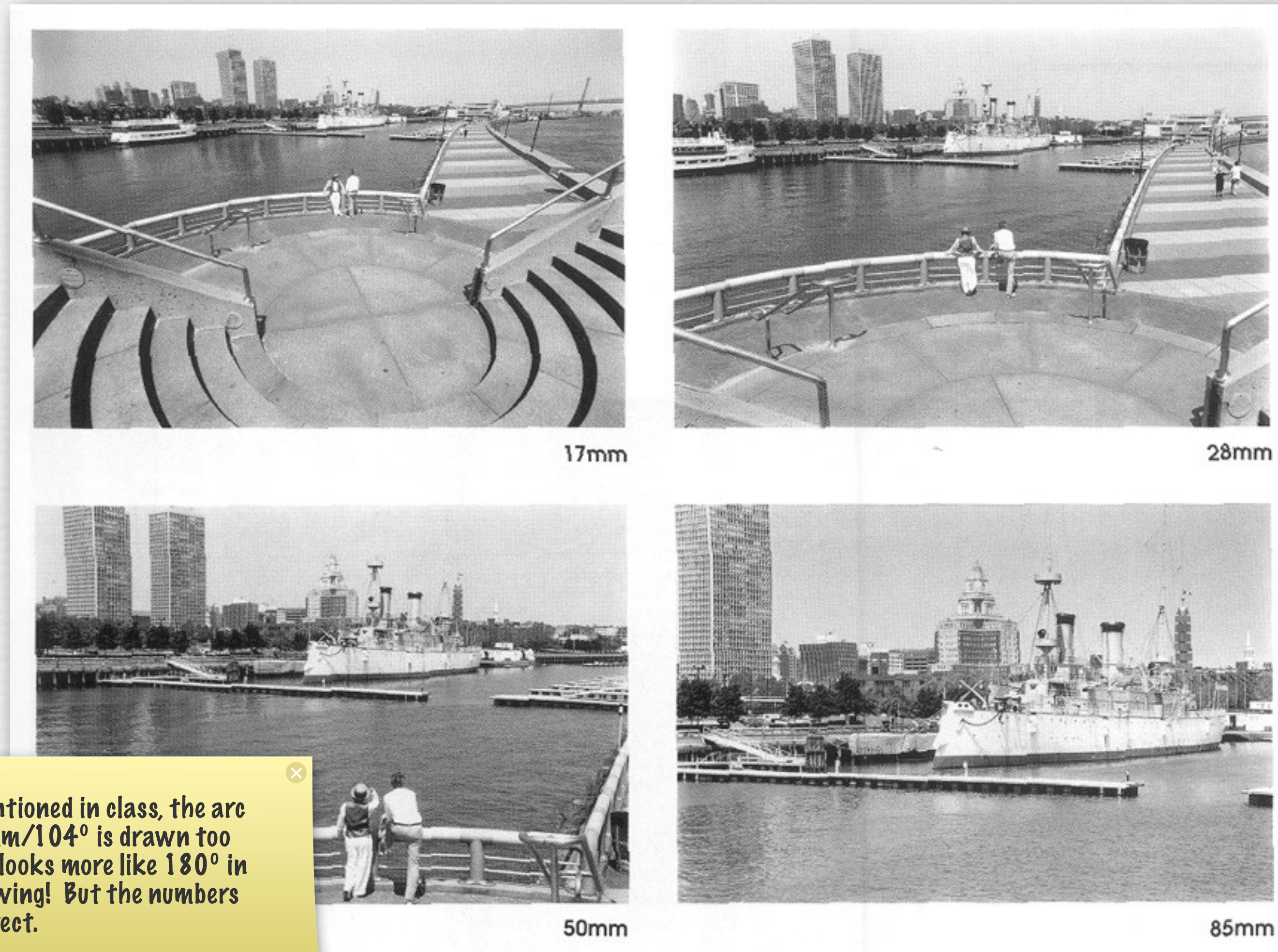


(Kingslake)

$$FOV = 2\ arctan\ (h\ /\ 2\ f\ )$$

Focal length and field of view



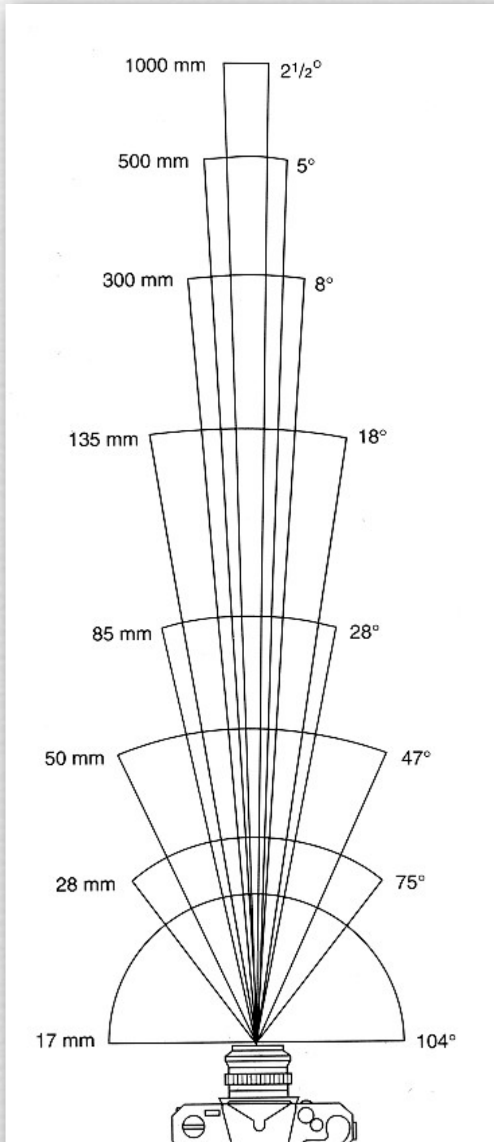


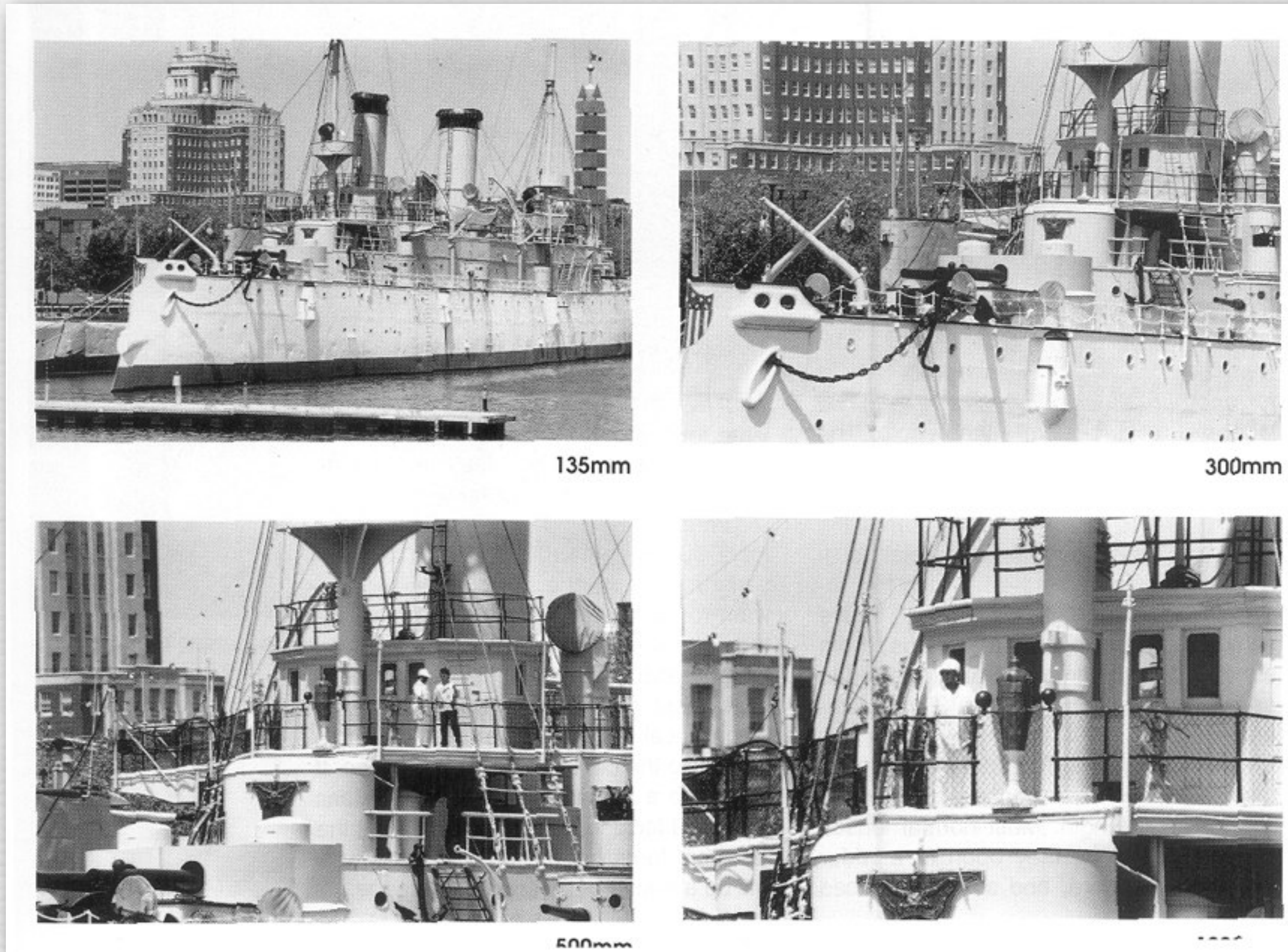
As I mentioned in class, the arc for 17mm/104° is drawn too wide; it looks more like 180° in the drawing! But the numbers are correct.

(London)

FOV measured diagonally on a 35mm full-frame camera (24 × 36mm)

Focal length and field of view





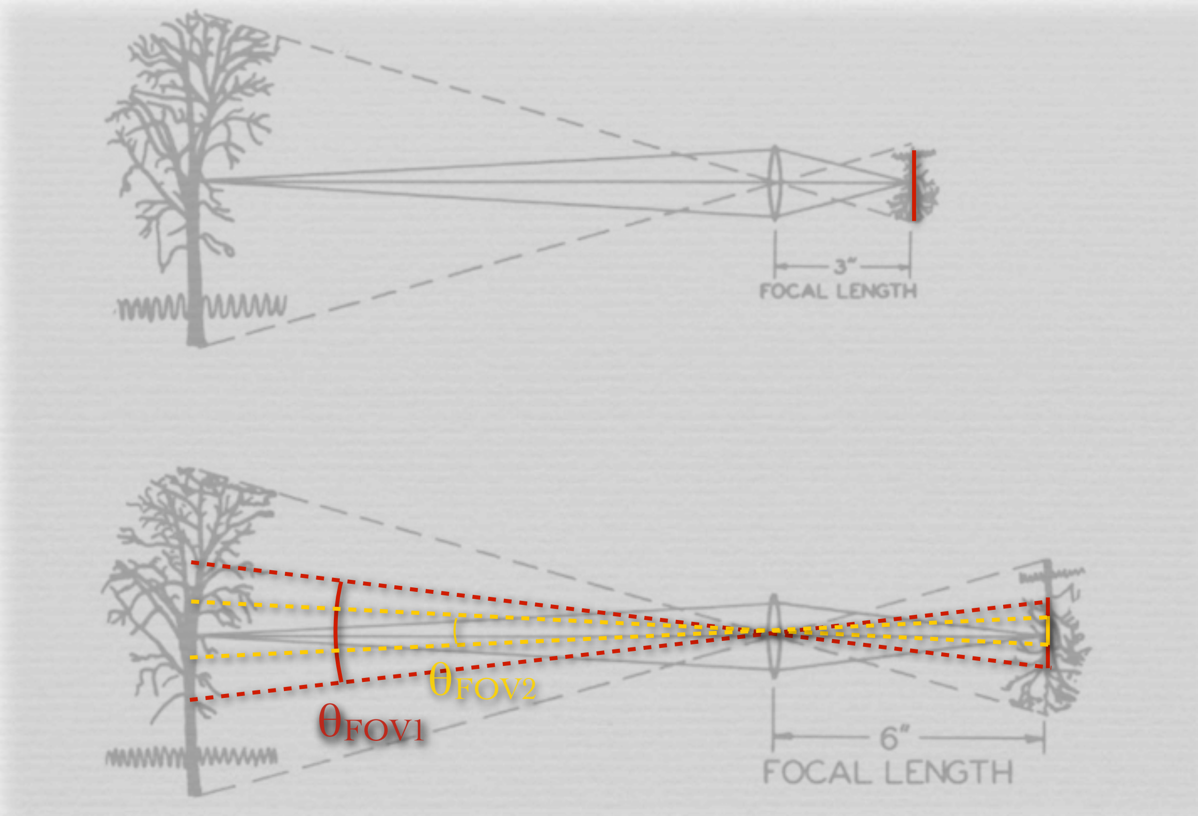
(London)

FOV measured diagonally on a
35mm full-frame camera (24 × 36mm)

Changing the sensor size

- if the sensor size is smaller, the field of view is smaller too

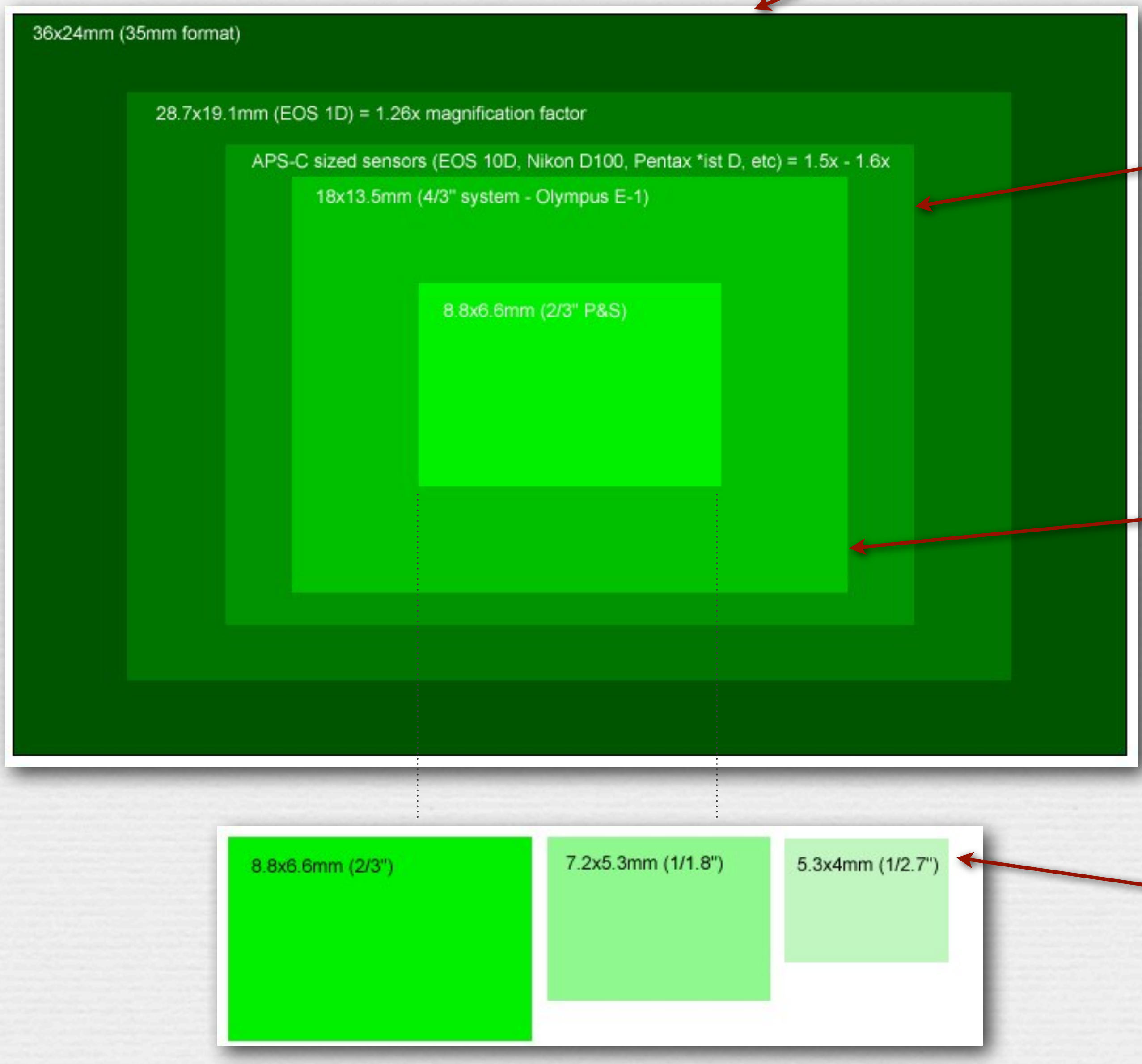
- smaller sensors either have fewer pixels, or noiser pixels



(Kingslake)

Sensor sizes

"full frame"
Canon 5D Mark II
(24mm × 36mm)



"APS-C"
Nikon D40
(15.5mm × 23.7mm)
(~1.5× crop factor)

"micro 4/3"
Panasonic GF1
(13mm × 17.3mm)
(~2× crop factor)

"point-and-shoot"
Canon A590
(5.75mm × 4.31mm)
(~8× crop factor)

Paris, 2009 (Panasonic GF1 + Leica 90mm)

Changing the focal length versus changing the viewpoint

(Kingslake)

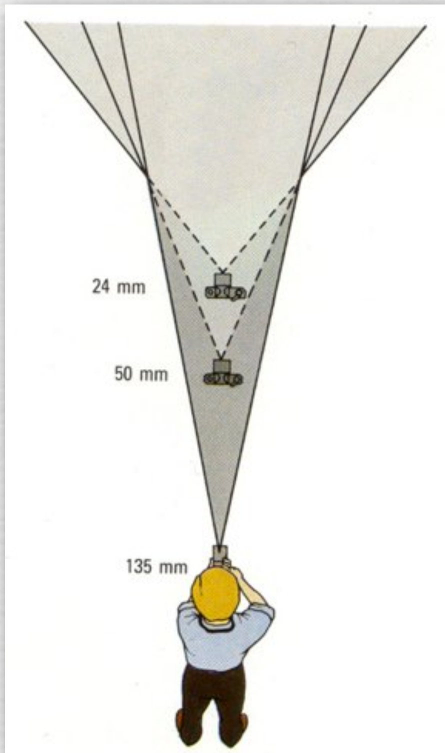
wide-angle

telephoto and moved back

- changing the focal length lets us move back from a subject, while maintaining its size on the image
- but moving back changes perspective relationships

Changing the focal length versus changing the viewpoint

- moving back while changing the focal length lets you keep objects at one depth the same size
- in cinematography, this is called the dolly zoom, or "Vertigo effect", after Alfred Hitchcock's movie



Effect of focal length on portraits

- standard “portrait lens” is 85mm

wide angle

standard

telephoto

Recap

- pinhole cameras compute correct linear perspectives
  - but dark
  - diffraction limited
- lenses gather more light
  - but only one plane of scene is in focus
  - focus by moving the sensor or lens
- focal length determines field of view
  - from wide angle to telephoto
  - depends on sensor size

more in the lens lectures next week

Questions?

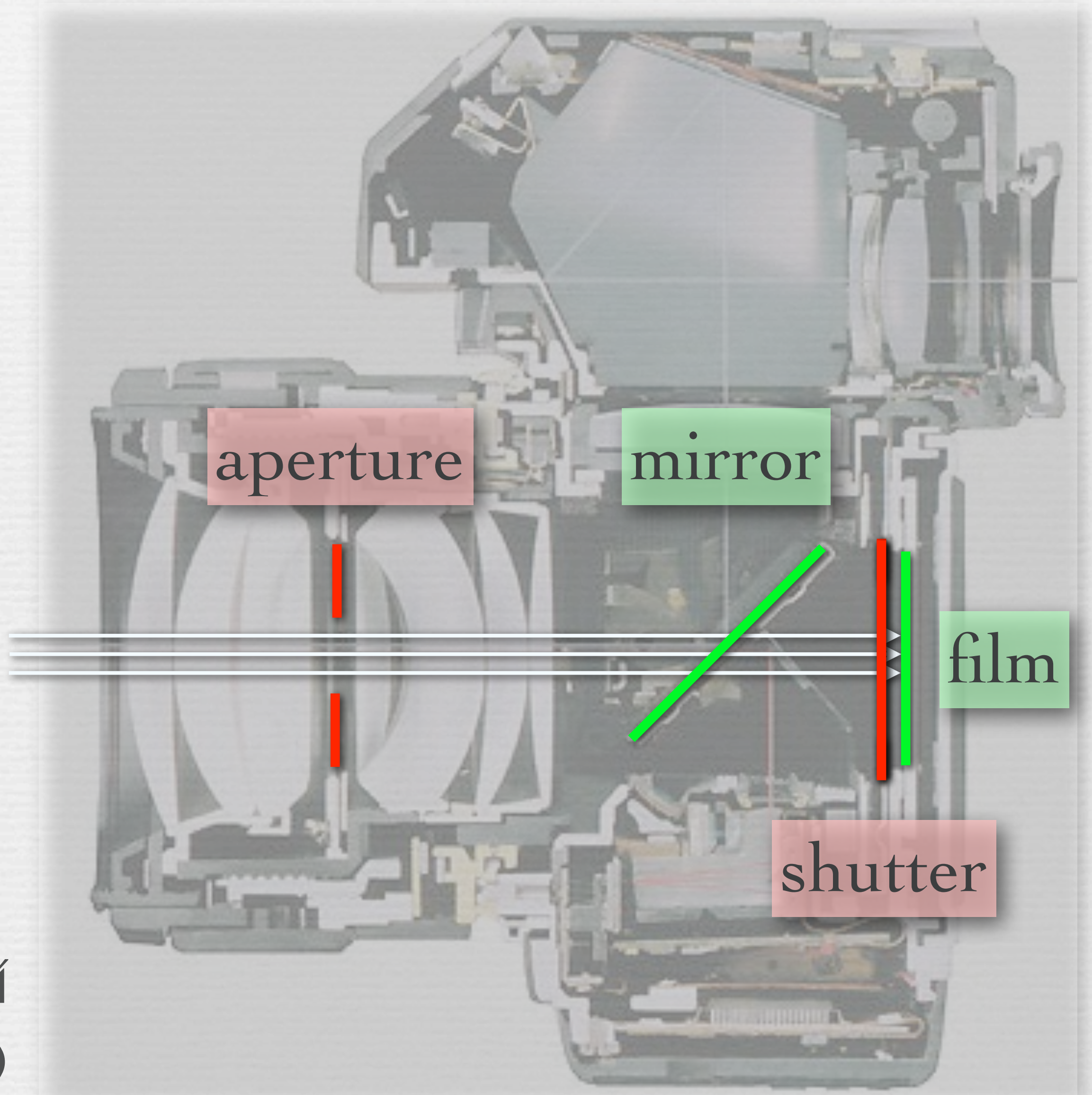
Exposure

- $H = E \times T$
- exposure = irradiance × time
- irradiance (E)
  - amount of light falling on a unit area of sensor per second
  - controlled by aperture
- exposure time (T)
  - in seconds
  - controlled by shutter

Single lens reflex camera

Nikon F4
(film camera)

Single lens reflex camera

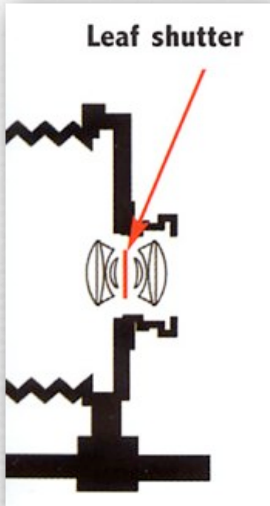


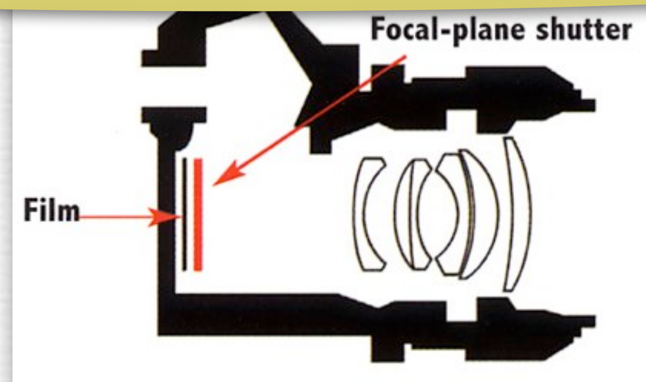
Nikon F4
(film camera)

Shutters

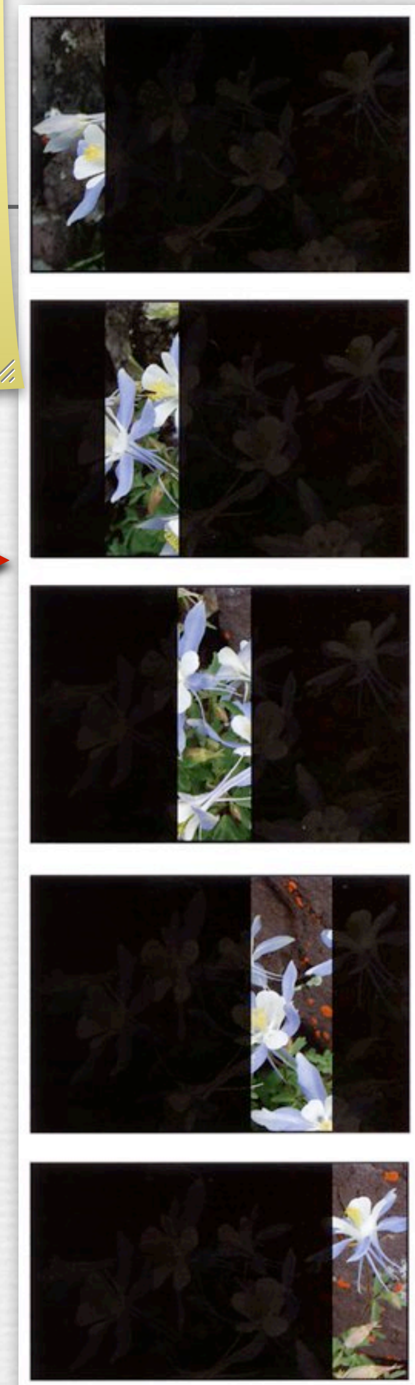
Since the leaf shutter is a long way from the film or sensor plane, it is out of focus. In fact, it is maximally out of focus. Thus, if it's a bit slow to open or close, meaning that the edges get less exposure time than the middle, it doesn't matter, since it's only the edges and center of the **aperture**, not the **field of view**. (Actually, any slowness in opening or closing will slightly change the bokeh - the shape of out-of-focus scene points; we'll talk more about bokeh next week.)

However, you couldn't put a leaf shutter near the film or sensor, because then it **would** be in focus, and its slow action would cause the edges of the field of view to get less light than the center. To avoid this problem, focal-plane shutters adopt the different slit-type action shown here. This action ensures that all parts of the field of view receive the same duration of exposure (even if not all at exactly the same moment).





(London)



- quiet
- slow (max 1/500s)
- out of focus
- need one per lens

- loud
- fast (max 1/4000)
- in focus
- distorts motion

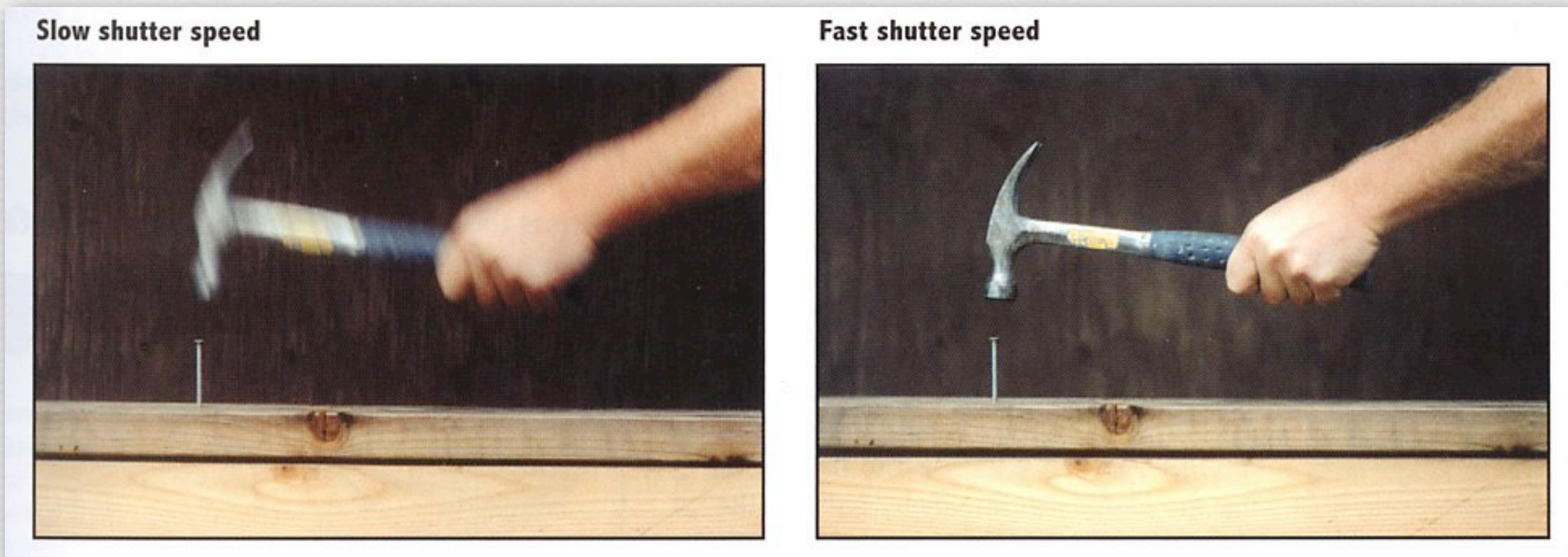
Jacques-Henri Lartigue, Grand Prix (1912)

Shutter speed

- controls how long the sensor is exposed to light
- linear effect on exposure until sensor saturates
- denoted in fractions of a second:
  - 1/2000, 1/1000,...,1/250, 1/125, 1/60,...,15, 30, B(ulb)
- normal humans can hand-hold down to 1/60 second
  - *rule of thumb:* shortest exposure = 1 / *f*
  - e.g. 1/500 second for a 500mm lens

Main side-effect of shutter speed

- motion blur
- halving shutter speed doubles motion blur

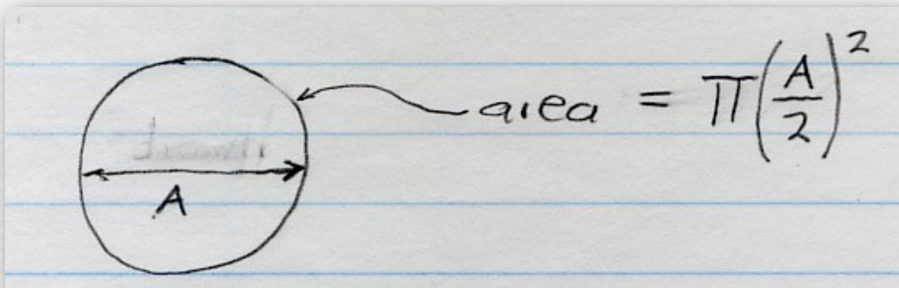


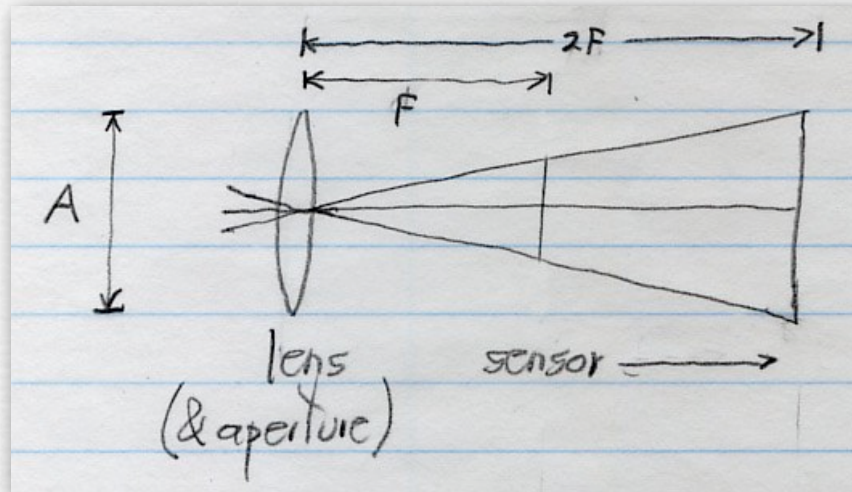
(London)

Aperture

- irradiance on sensor is proportional to
  - square of aperture diameter A
  - inverse square of distance to sensor (~ focal length f)

Irradiance on sensor (contents of whiteboard)





- As the diameter A of the aperture doubles, its area (hence the light that can get through it) increases by 4× (first drawing).
- Think of the lens as a collection of pinholes, each having a fixed angular field of view (cone in 2nd drawing) determined by the lens design.
- A certain amount of light gets through each pinhole. By conservation of energy, that light will fall on whatever sensor is placed in its path.
- If the distance to the sensor is doubled, the area intersecting the cone increases by 4×, so the light falling per unit area decreases by 4×.

Aperture

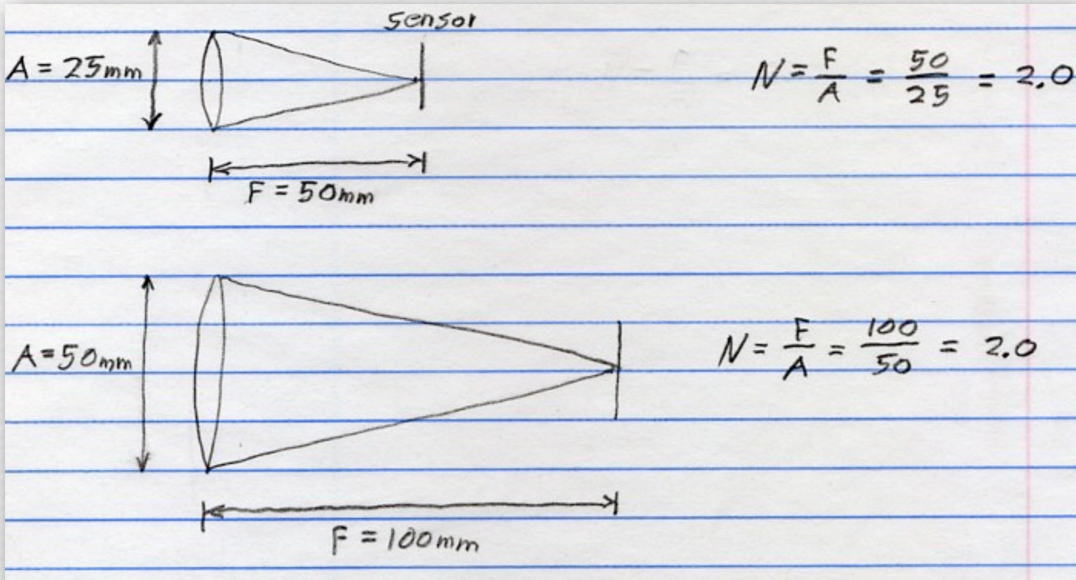
- irradiance on sensor is proportional to
  - square of aperture diameter A
  - inverse square of distance to sensor (~ focal length f)
- so that aperture values give irradiance regardless of lens, *aperture number* N is defined relative to focal length

$$N = \frac{f}{A}$$

  - f/2.0 on a 50mm lens means the aperture is 25mm
  - f/2.0 on a 100mm lens means the aperture is 50mm

  ∴ low F-number (N) on long telephotos require fat lenses

Example F-number calculations (contents of whiteboard)



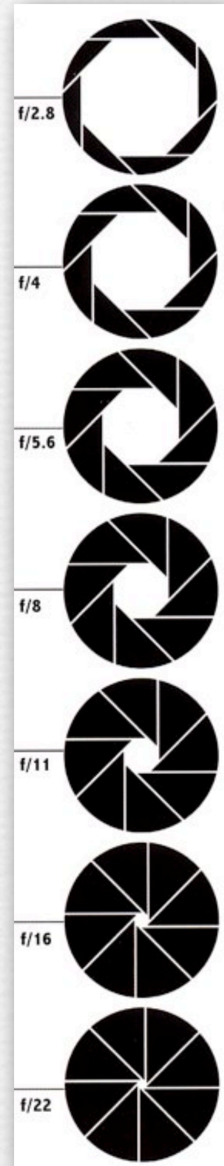
- A relative aperture size (called F-number or just N) of 2 is sometimes written f/2, reflecting the fact that it is computed by dividing focal length (f) by the absolute aperture diameter (A).
- As this drawing shows, doubling both the absolute aperture diameter (A) and the focal length (f) cancel; leaving the same relative aperture size (N). In this example, both lenses are f/2.

Aperture

- irradiance on sensor is proportional to
  - square of aperture diameter A
  - inverse square of distance to sensor (~ focal length f)
- so that aperture values give irradiance regardless of lens, *aperture number* N is defined relative to focal length

$$N = \frac{f}{A}$$

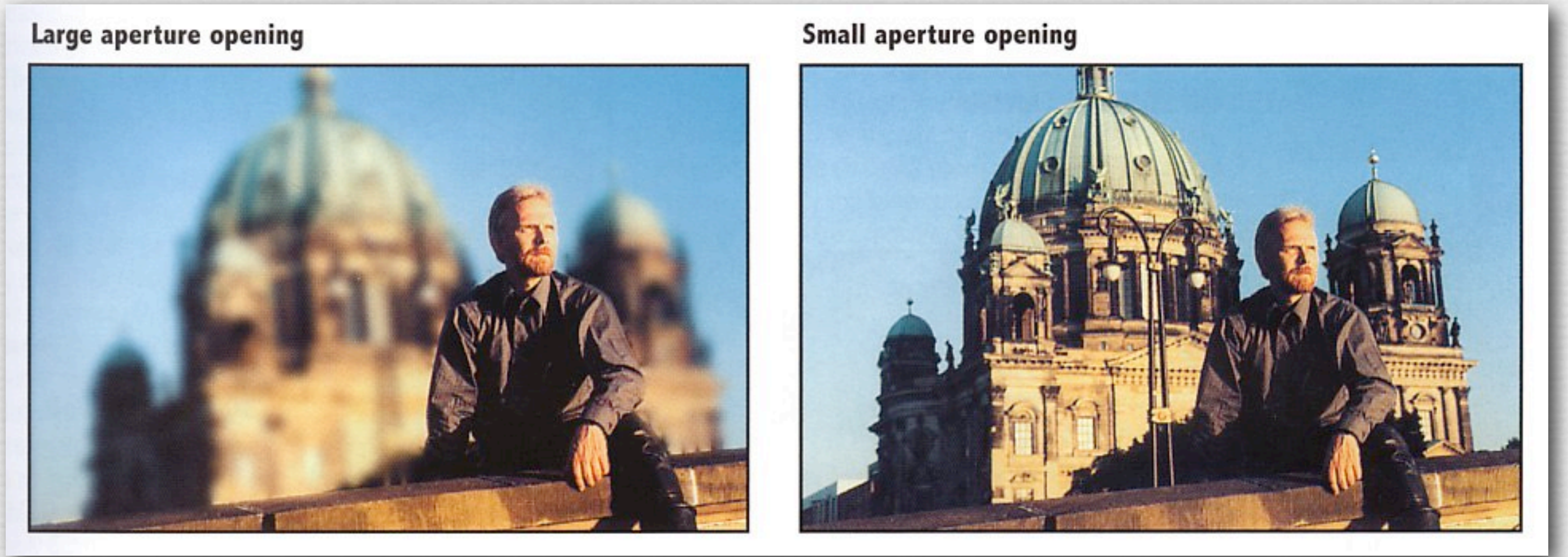
  - f/2.0 on a 50mm lens means the aperture is 25mm
  - f/2.0 on a 100mm lens means the aperture is 50mm
  - ∴ low F-number (N) on long zooms require fat lenses
- doubling N reduces A by 2×, hence light by 4×
  - going from f/2.0 to f/4.0 cuts light by 4×
  - to cut light by 2×, increase N by $\sqrt{2}$



(London)

Main side-effect of aperture

- depth of field
- doubling N (two f/stops) doubles depth of field



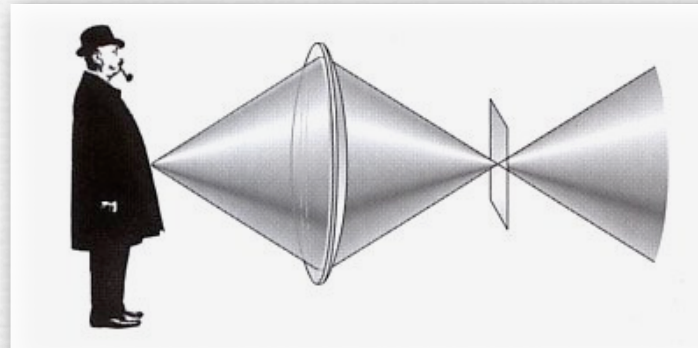
(London)

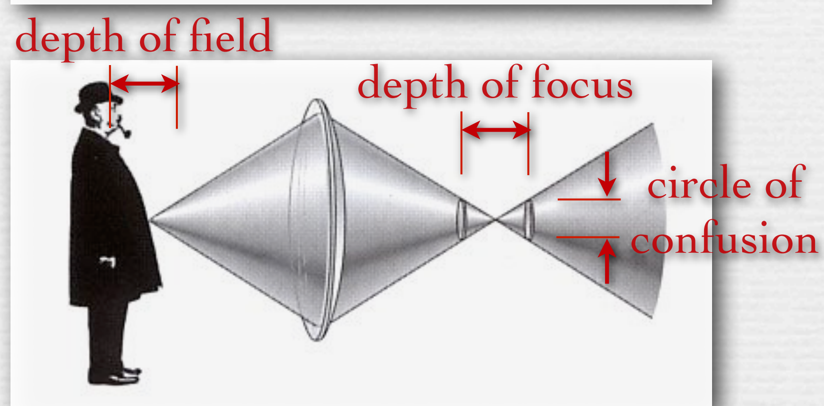
Depth of field (briefly)

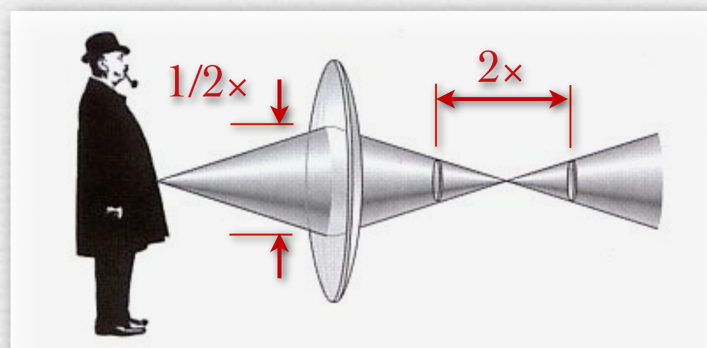
This figure isn't quite right; we'll fix it next week

- a point in the scene is focused at a point on the sensor
- if we move the sensor in depth, the point becomes blurred
- if it blurs too much, it exceeds our allowable *circle of confusion*
- the depth where this happens is called the *depth of focus*
- this corresponds in the scene to a *depth of field*
- halving the aperture diameter doubles the depth of field

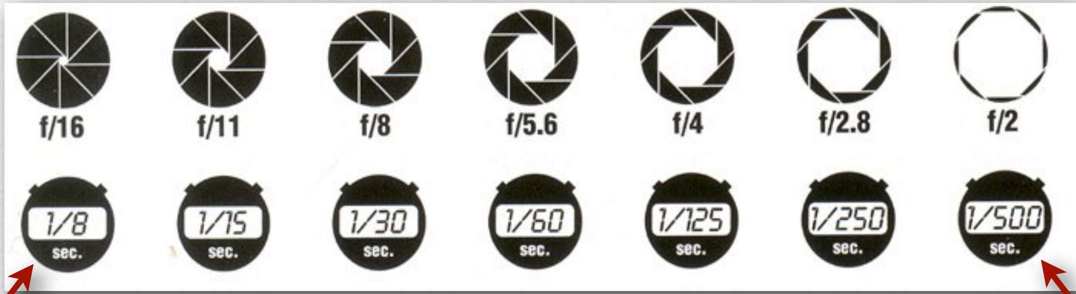
(London)

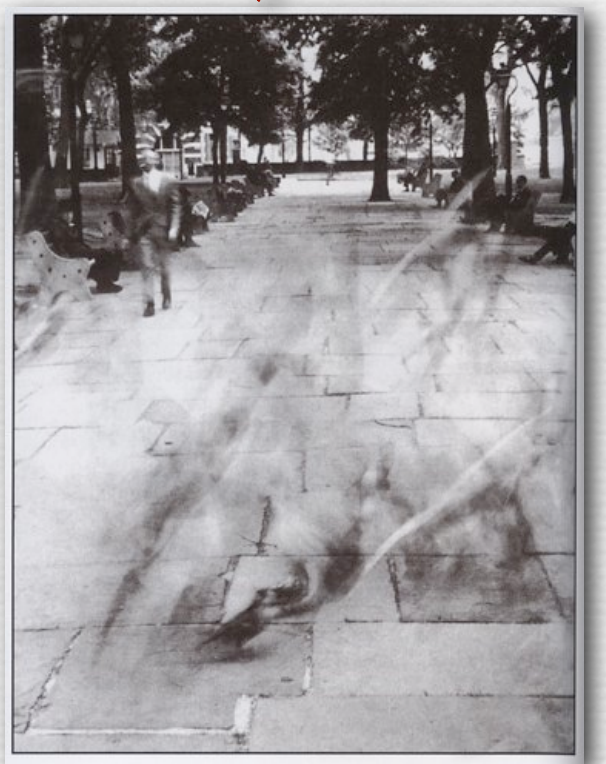


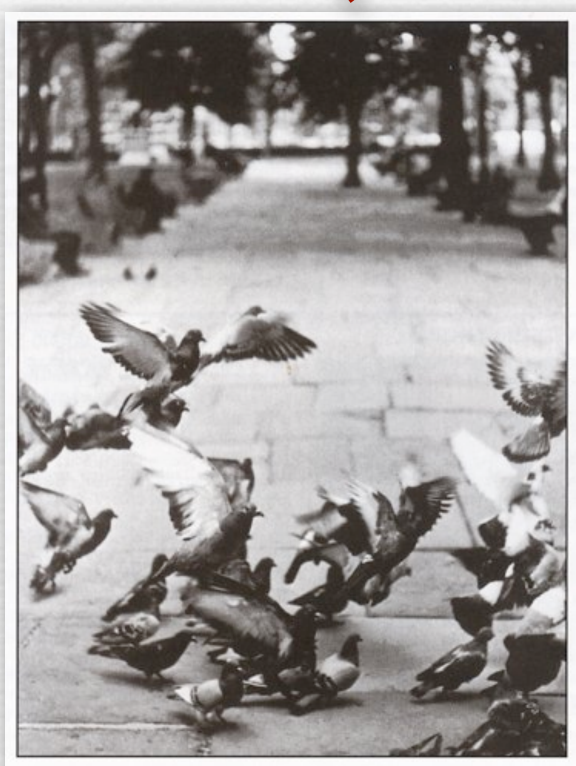




Trading off motion blur and depth of field







(London)

Recap

(London)

- ✦ H = E × T
- ✦ exposure = irradiance × time
- ✦ irradiance (E)
  - controlled by the aperture
  - lowering by one f/stop doubles H
  - lowering by two f/stops doubles depth of field
- ✦ exposure time (T)
  - controlled by the shutter
  - doubling exposure time doubles H
  - doubling exposure time doubles motion blur

Questions?

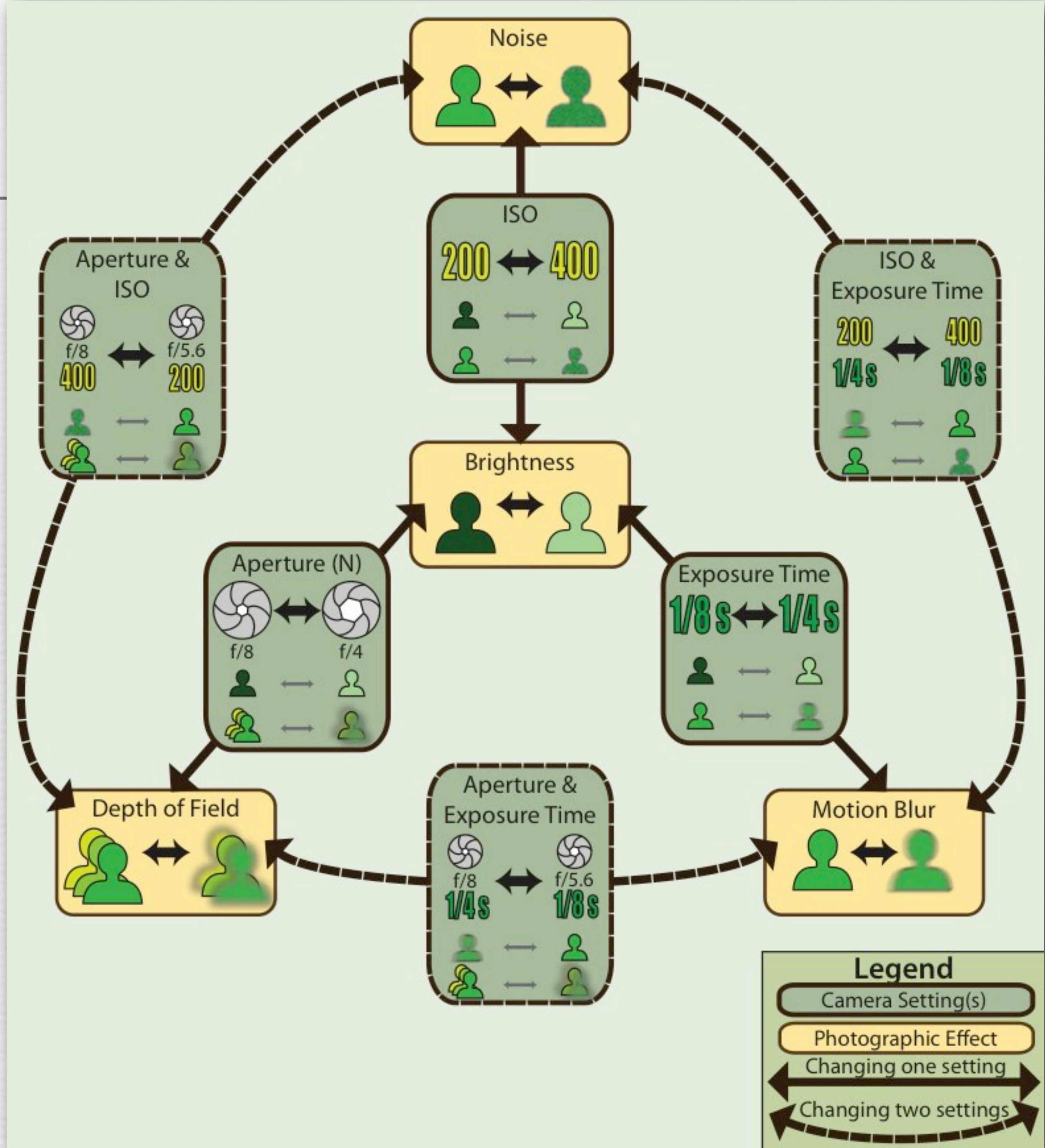
Sensitivity (ISO)

- third variable for exposure
- film: trade sensitivity for grain
- digital: trade sensitivity for noise
    - multiply signal before analog-to-digital conversion
    - linear effect (200 ISO needs half the light as 100 ISO)

more in noise lecture

(dpreview)

Tradeoffs affecting brightness



(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178-10/applets/exposure.html

(Eddy Talvala)

Slide credits

- Steve Marschner
- Fredo Durand
- Eddy Talvala

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